Faculty of Physics, University of Vienna &
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Quantum Causal Structures

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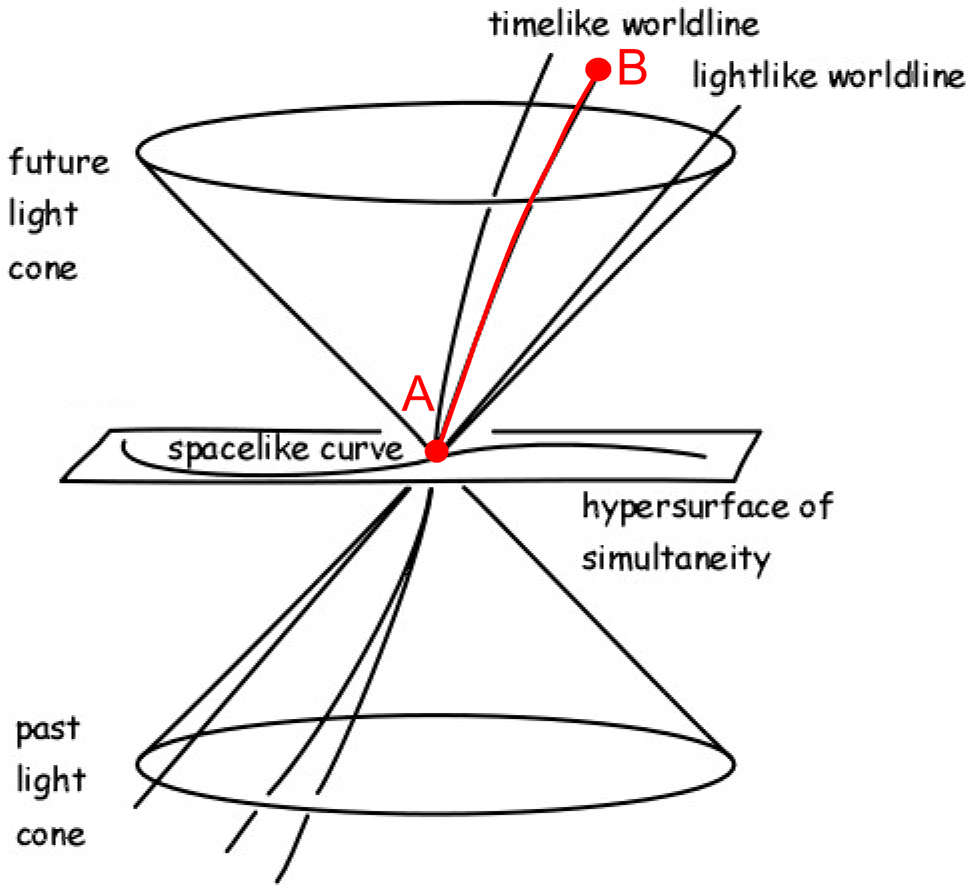
Philippe Allard Guérin, Adrien Feix, Ognyan Oreshkov, Igor Pikovski,
Magdalena Zych, Časlav Brukner

Quantum Spacetime '19
Bratislava, 11-15 February 2019

Definite causal order
(general relativity)

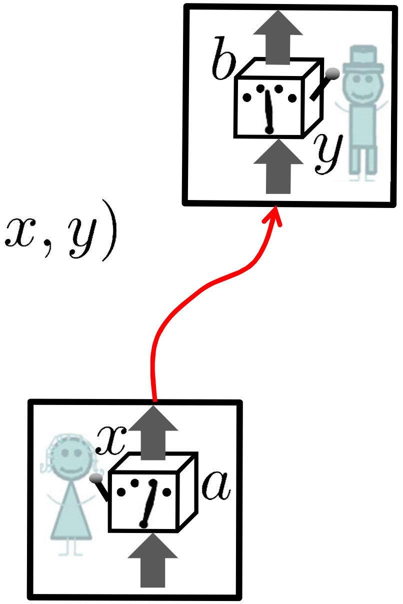


In all our theories (quantum gravity theories excluded) **causal relations are definite**:

An event B is contained in the causal future of an event A if there exists a future-directed space-time path connecting A to B such that the tangent vector to this path is everywhere time-like or null.

Picture taken from: https://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/spacetime/

Definite causal order
(quantum information)



$p(a,b|x,y)$

$$\sum_a p(a,b|x,y) = p(b|x,y)$$

$$\sum_b p(a,b|x,y) = p(a|x)$$

Causal relation is definite:
A can signal to B, but B cannot signal to A

One-directional signalling
“from the past to the future”

Č. B., Nature Physics **10**, 259–263 (2014).
C. Branciard, M. Araújo, F. Costa, A. Feix, and Č. B., New J. Phys. **18**, 013008 (2016).

„ … *once we embark on constructing a quantum theory of gravity, we expect some sort of quantum fluctuations in the metric, and so also in the causal structure. But in that case, how are we to formulate a quantum theory with a fluctuating causal structure*?”

Butterfield and Isham, in Physics Meets Philosophy at the Planck Scale: Contemporary Theories in Quantum Gravity, C. Callender and N. Huggett, eds. Cambridge University Press, 2001.

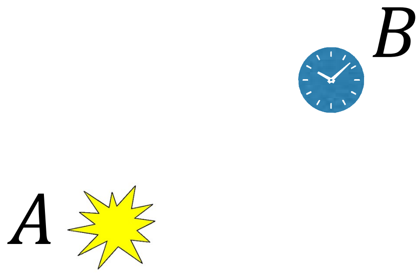
Towards Quantum Gravity: A Framework for Probabilistic Theories with Non-Fixed Causal Structure

Lucien Hardy
Perimeter Institute,
31 Caroline Street North,
Waterloo, Ontario N2L 2Y5, Canada

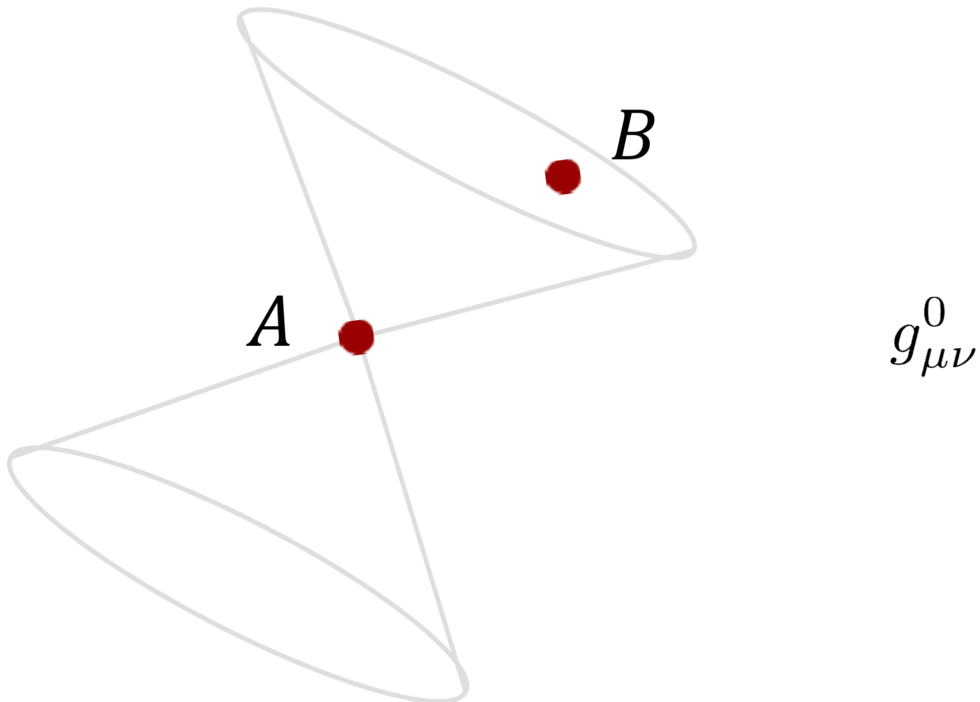
February 4, 2008

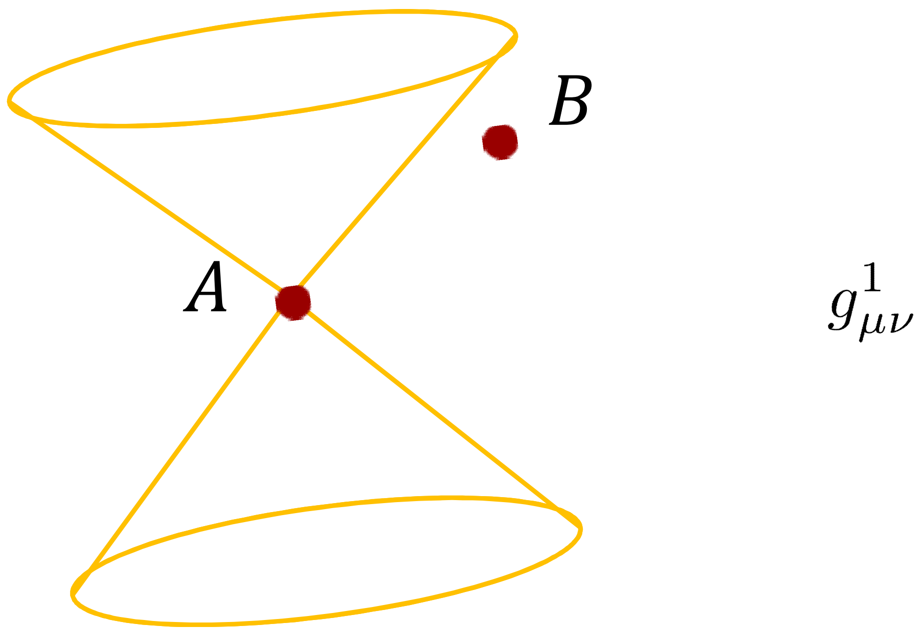
Abstract

General relativity is a deterministic theory with non-fixed causal structure. Quantum theory is a probabilistic theory with fixed causal structure. In this paper we build a framework for probabilistic theories with non-fixed causal structure. This combines the radical elements of general relativity and quantum theory. We adopt an operational methodology for the purposes of theory construction (though without committing to operationalism as a fundamental philosophy). The key idea in the con-

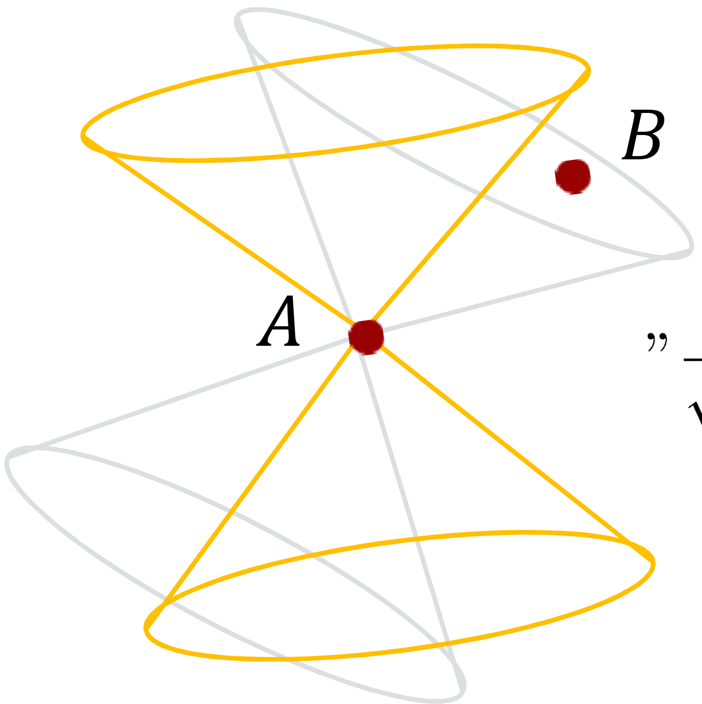


Events specified operationally, without explicitly relying on a background classical spacetime. („diffeomorphism invariance“)


B
A
$g^{0}_{\mu\nu}$

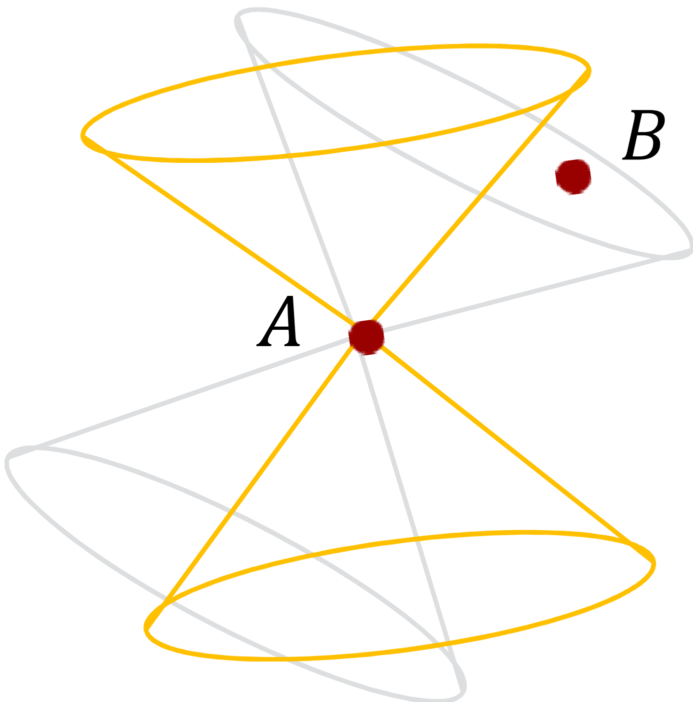

A
B
$g^1_{\mu\nu}$

“Superpositions of causal structures”?



$$“\frac{1}{\sqrt{2}}(|g^0_{\mu\nu}\rangle + |g^1_{\mu\nu}\rangle)”$$

“Superpositions of causal structures”?



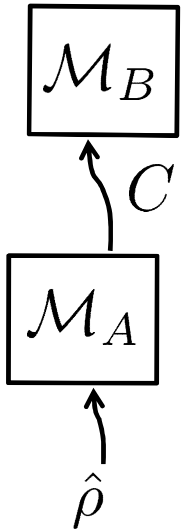
$|0\rangle$ $|1\rangle$

$$\text{"}\,\frac{1}{\sqrt{2}}(|0\rangle|g^{0}_{\mu\nu}\rangle + |1\rangle|g^{1}_{\mu\nu}\rangle)\text{"}$$

1957 Chapel Hill Conference, Richard Feynman

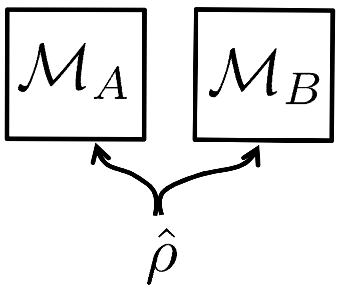
A problem

In standard formulation of quantum theory, time-like (& light-like) and space-like separated scenarios are mathematically described in **very different ways**.



Time-like separation

$$\mathcal{M}_B \circ C \circ \mathcal{M}_A(\rho)$$



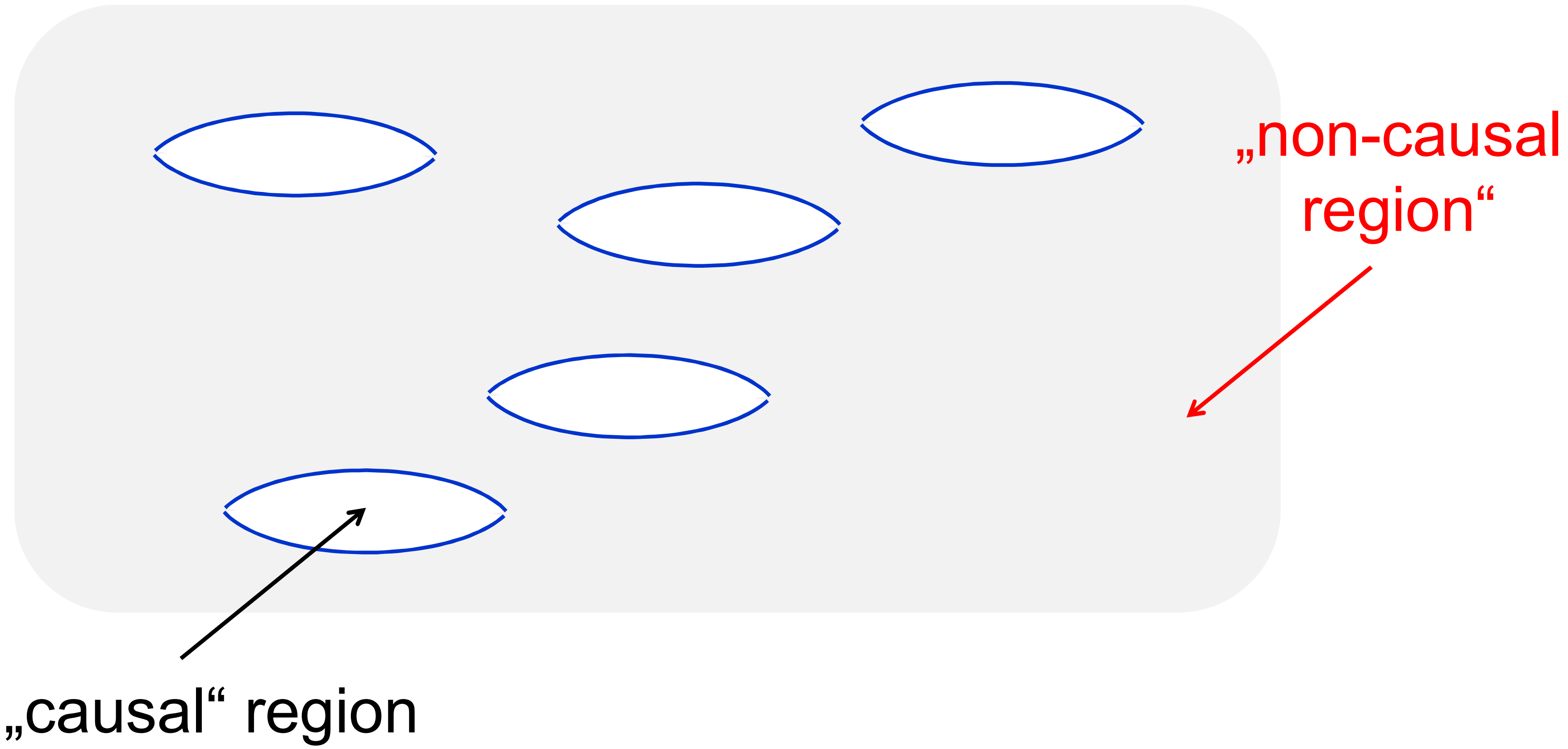
Space-like separation

$$(\mathcal{M}_A \otimes \mathcal{M}_B)(\rho)$$

Outline

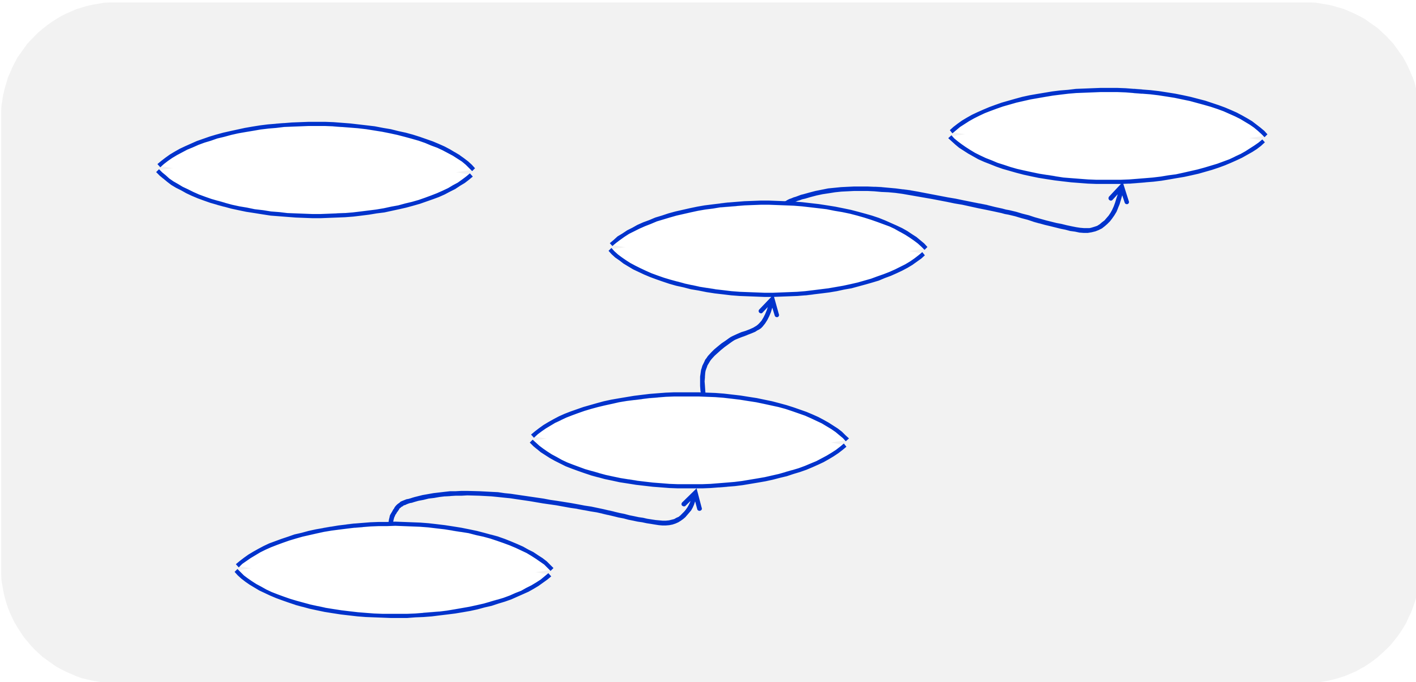
- The notion of event & causality
- Framework for quantum mechanics with no global causal structure:
  Causally non-separable processes ("indefinite causal order")
  Causal inequalities
  The quantum switch
- Advantages in quantum computation and communication
  (not this talk)
- Physical realization of causally non-separable processes
  via superposition of large masses

Intuitive picture



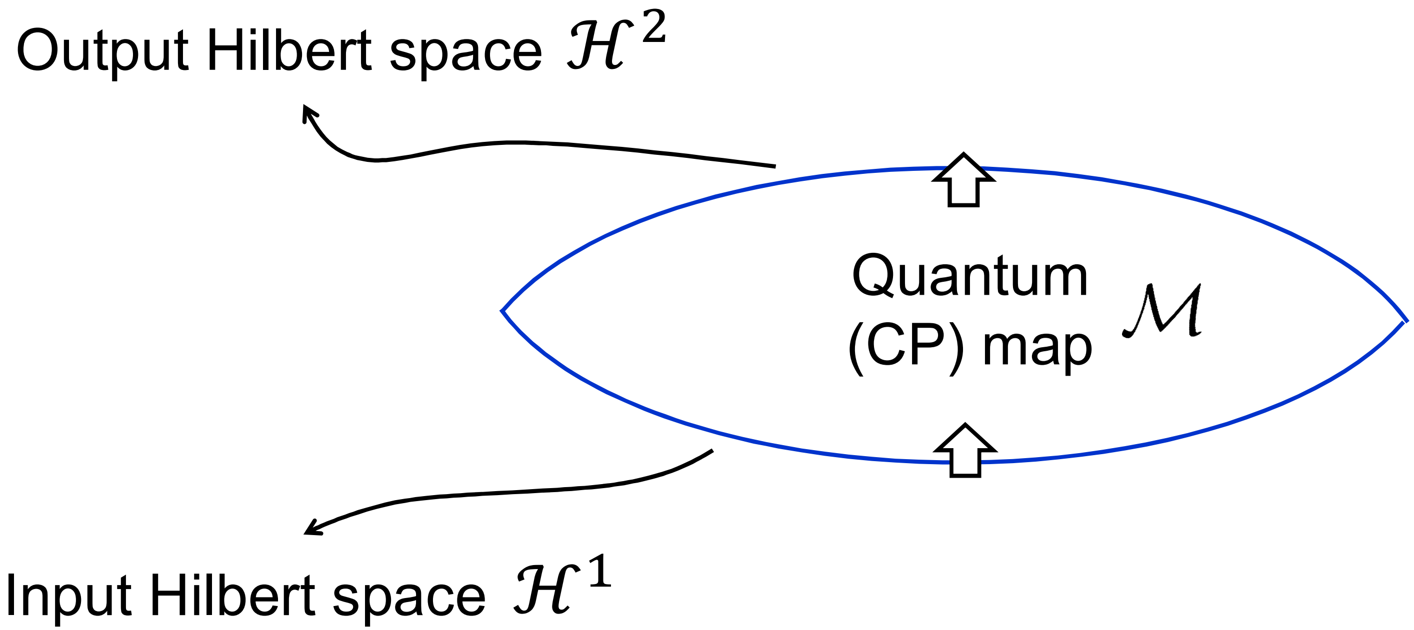
Locally causal, globally indefinite

Intuitive picture

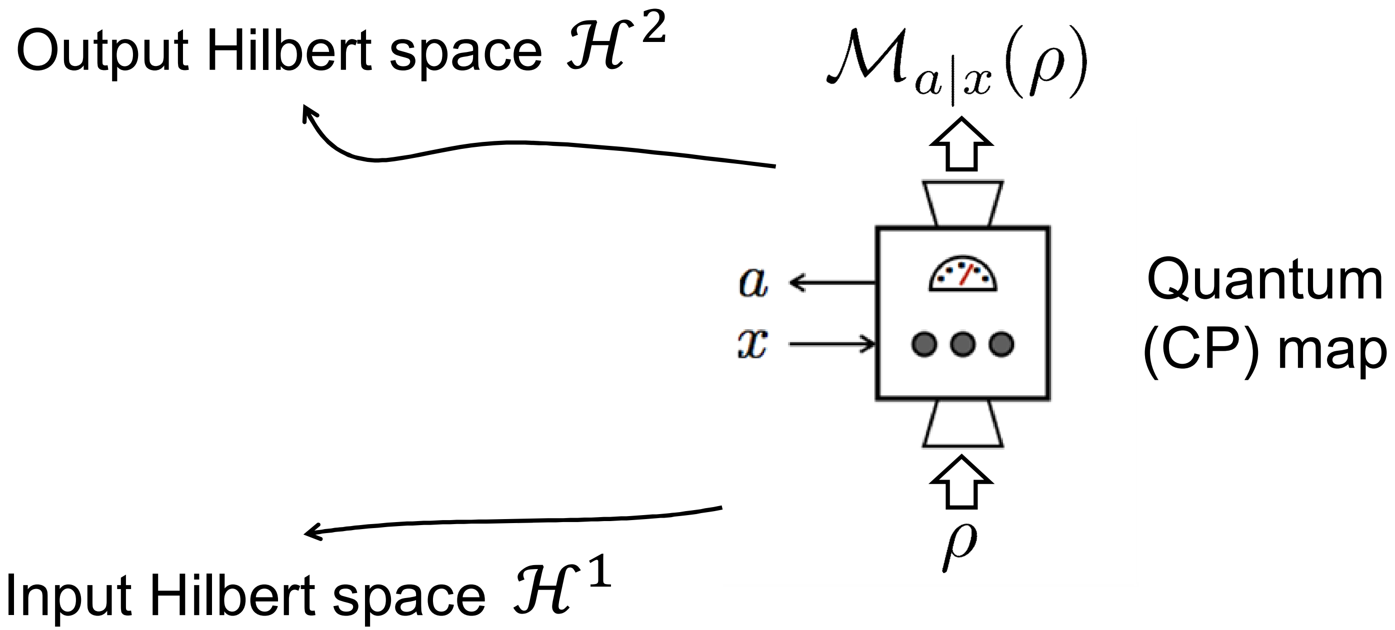


Globally causal

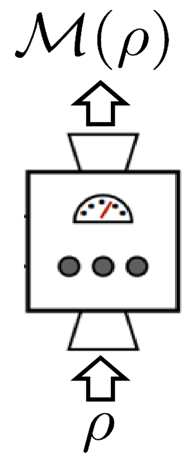
Local space-time patch



Operational view: Local quantum laboratory



The Choi-Jamilolkowski isomorphism



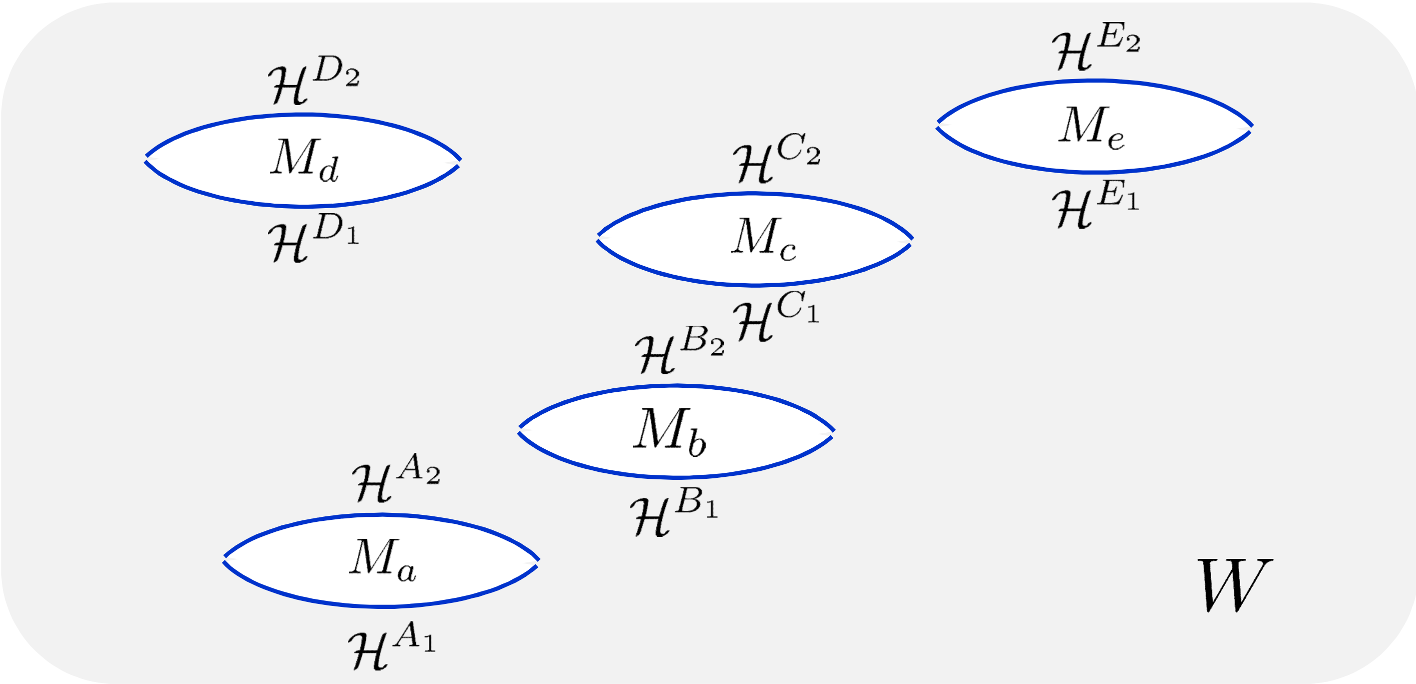
$$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2) \quad \Leftrightarrow \quad M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$$

$$M = (\mathcal{M} \otimes \mathbb{1})(|\Phi^+\rangle\langle\Phi^+|) = \sum_{ij} |i\rangle\langle j|^1 \otimes \mathcal{M}(|i\rangle\langle j|)^2$$

where $|\Phi^+\rangle = \sum_j |j\rangle_1 |j\rangle_2$

Inverse isomorphism: $\mathcal{M}(\rho) = \mathrm{Tr}_1[M(\rho^T \otimes \mathbb{1})]$

General quantum correlations



Generalized Born's rule:

Local maps, describe the interiors of the labs

$$p(a, b, c, d, \ldots) = \mathrm{Tr}[W(M_a \otimes M_b \otimes M_c \otimes M_d \otimes \ldots)]$$

Process matrix, describe the causal relations between the labs

Characterisation of processes

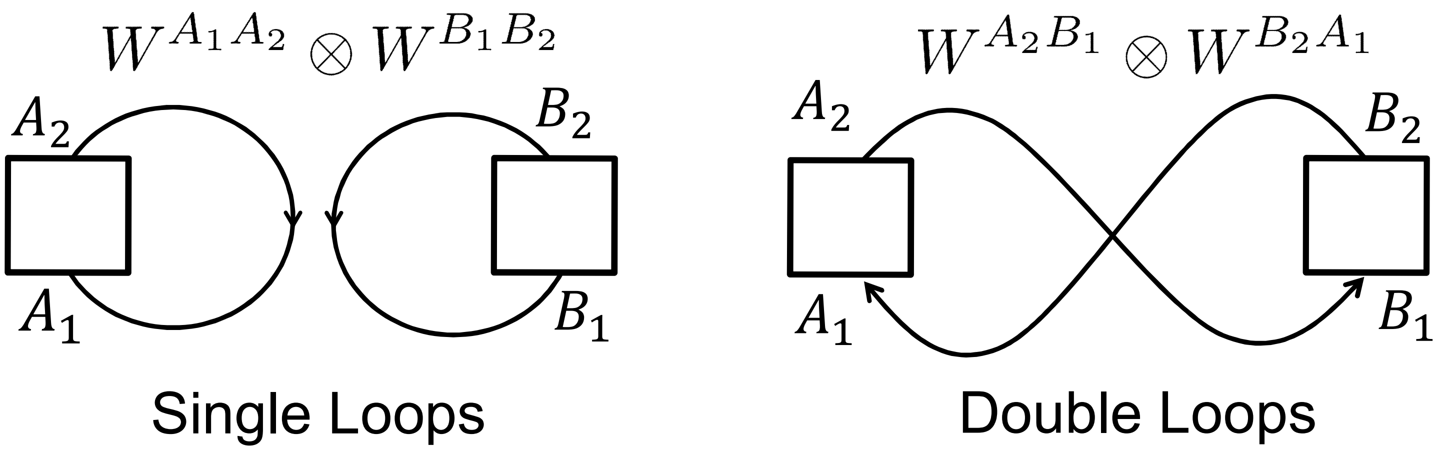
Theorem: The positivity and normalisation of the probabilities imply

$$W \geq 0, \quad W = \mathcal{L}_V W$$

↑

Projection onto a subspace of process matrices with **no causal loops**

Forbidden processes (producing the grandfather paradox):

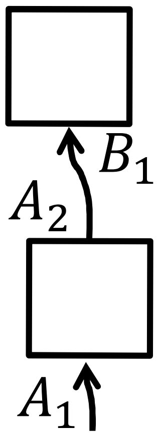


O. Oreshkov, F. Costa, Č.B., Nature Communication **3**: 1092 (2012).

Characterisation of processes

Allowed processes:

$$W^{A_1A_2B_1} \otimes \mathbb{1}^{B_2}$$



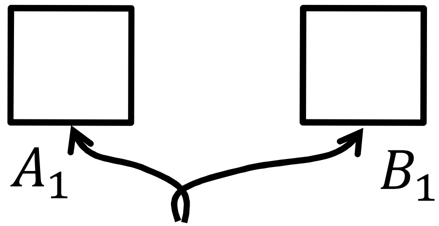
Channel from A to B

Time-like separation

Example of a channel:

$$|\psi\rangle^{A_1}|\mathbb{1}\rangle\rangle^{A_2B_1} = |\psi\rangle^{A_1}\sum_j |j\rangle^{A_2}|j\rangle^{B_1}$$

$$W^{A_1B_1} \otimes \mathbb{1}^{A_2B_2}$$



States

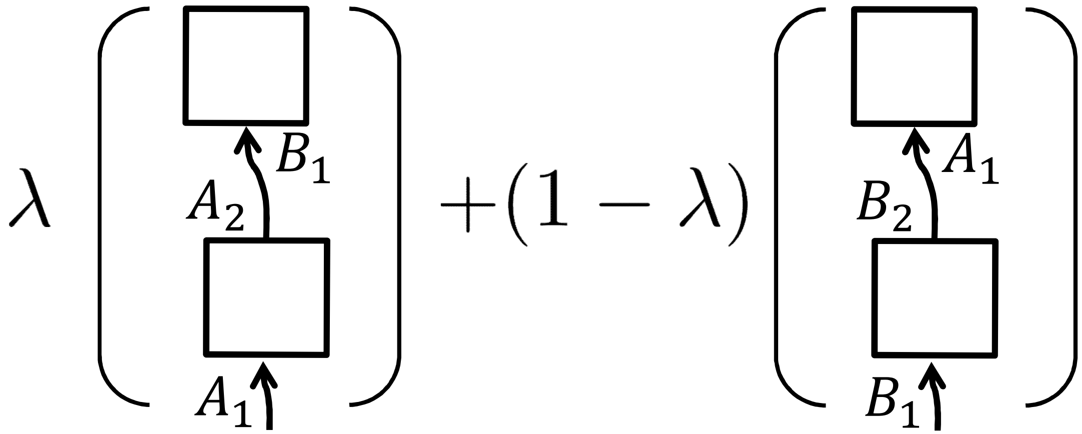
Space-like separation

Process matrix formalism is a **unified quantum framework** to describe space-like and time-like separated scenarios.

O. Oreshkov, F. Costa, Č.B., Nature Communication **3**: 1092 (2012).

Causally separable processes

Most general processes compatible with definite causal structure (convex mixtures of ordered processes):



Channel from A to B Channel from B to A

$$W = \lambda W^{A \preceq B} + (1-\lambda) W^{B \preceq A}$$

O. Oreshkov, F. Costa, Č.B., Nature Communication **3**: 1092 (2012)

Causally non-separable processes

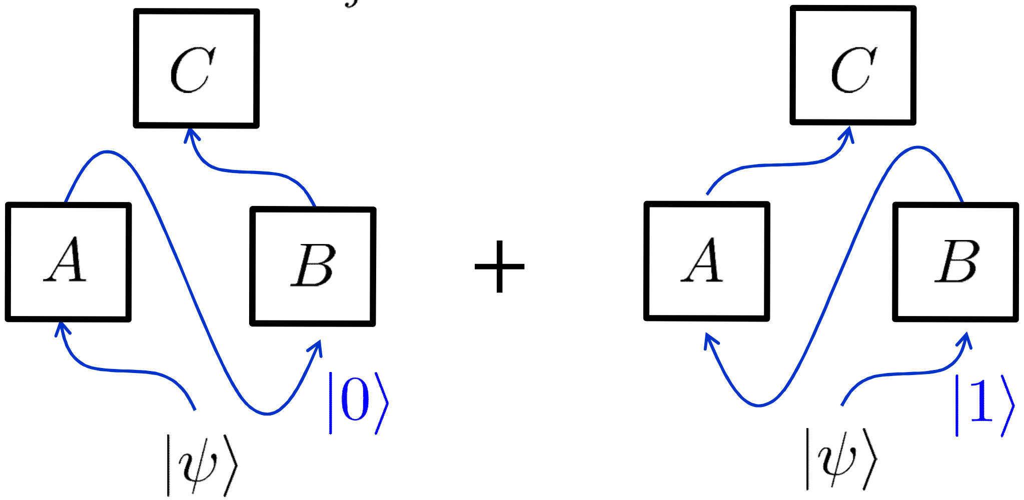
Theorem: The quantum switch is a **causally non-separable** process, i.e.

$$W \neq \lambda W^{A\preceq B\preceq C} + (1-\lambda) W^{B\preceq A\preceq C}$$

Example: **The quantum switch**

$$|W\rangle = |0\rangle^{cnt}|\psi\rangle^{A_1}|\mathbb{1}\rangle\rangle^{A_2B_1}|\mathbb{1}\rangle\rangle^{B_2C_1} + |1\rangle^{cnt}|\psi\rangle^{B_1}|\mathbb{1}\rangle\rangle^{B_2A_1}|\mathbb{1}\rangle\rangle^{A_2C_1}$$

Identity channel: $|\mathbb{1}\rangle\rangle = \sum_j |j\rangle|j\rangle$



G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, Phys. Rev. A **88**, 022318 (2013)
M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi and Č. B., New J. Phys. **17**, 102001 (2015)
O. Oreshkov and C. Giarmatzi, New J. Phys. **18**, 093020 (2016)

Creating causally non-separable processes

Gravitational time-dilation

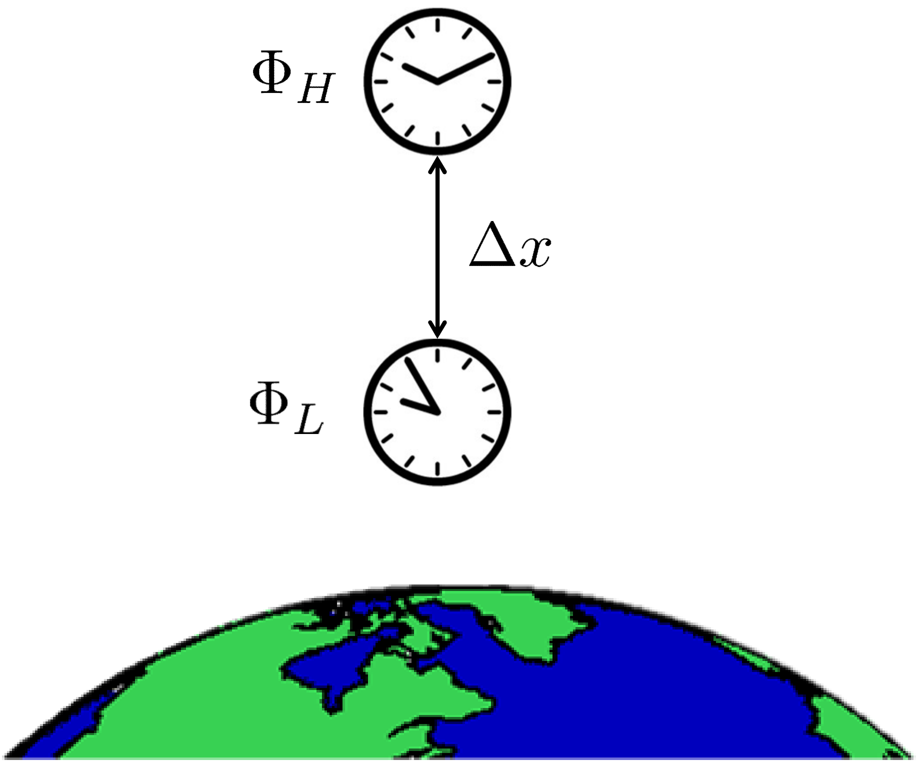
Initially synchronized clocks will eventually show **different times** when placed at different gravitational potentials.

Clock closer to a massive body ticks slower than the clock further away from the mass.

Gravitational time-dilation

Stationary metric, weak-field approximation: $g_{00} \approx -(1 + 2\frac{\Phi(x)}{c^2})$

$$g_{rr} \approx (1 + 2\frac{\Phi(x)}{c^2})^{-1}$$

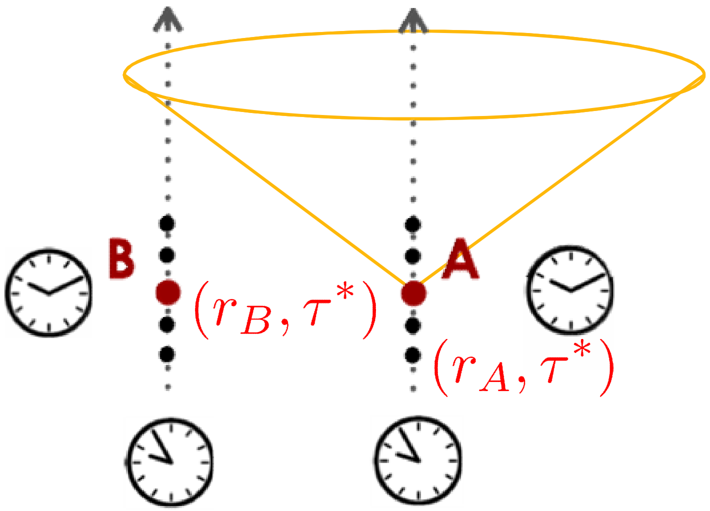


$$\frac{\Delta\tau_L}{\Delta\tau_H} = 1 - \sqrt{\frac{g_{00}(H)}{g_{00}(L)}} =$$

$$= 1 - \frac{\Phi_H - \Phi_L}{c^2} = 1 - \frac{g\Delta x}{c^2}$$

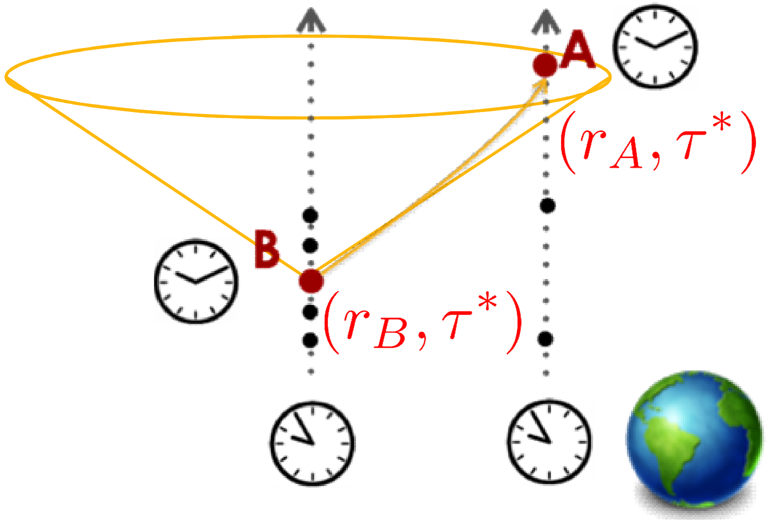
„lower clock is slower“

GR: Dynamical causal structure



The events A and B are space-like separated.

GR: Dynamical causal structure



Proper times at A and B

$$\tau_A = \sqrt{\frac{g_{00}(r_A)}{g_{00}(r_B)}}\tau_B$$

Coordinate time of photon propagation between A and B

$$T_c = \frac{1}{c}\int_{r_B}^{r_A} dr' \sqrt{-\frac{g_{rr}(r')}{g_{00}(r')}}$$

Event at A measured by A

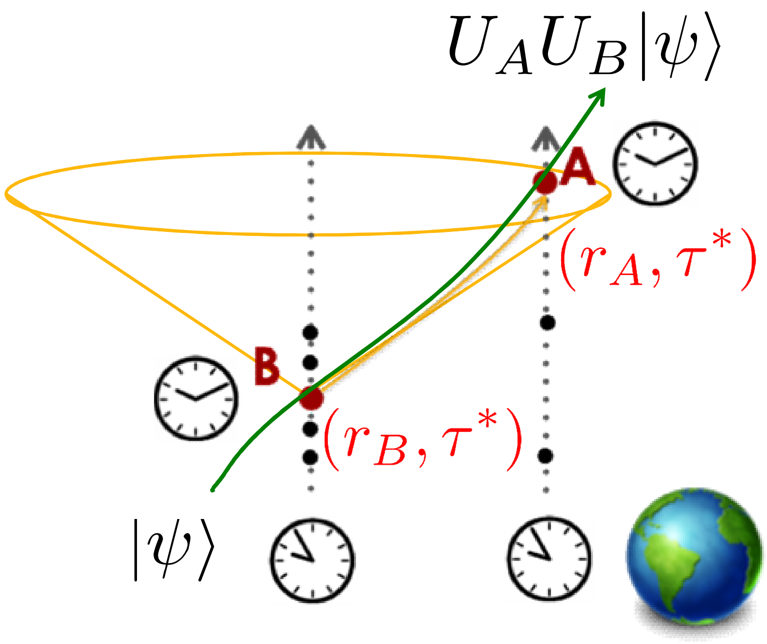
Event at B measured by B

$$\tau^* \geq \sqrt{\frac{g_{00}(r_A)}{g_{00}(r_B)}}\tau^* + T_c\sqrt{g_{00}(r_A)}$$

Event at B measured by A

Photon's propagation time measured by A

GR: Dynamical causal structure

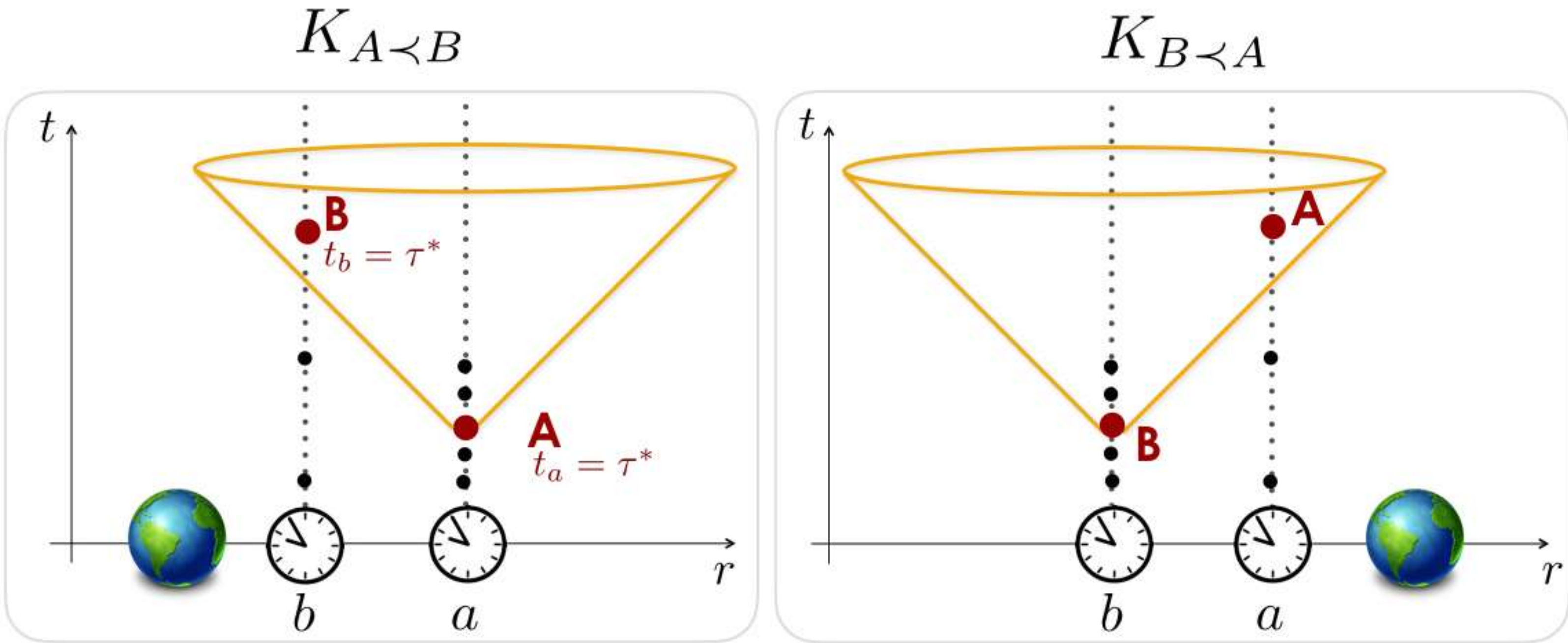


Mass configuration $K_{B \prec A}$

Channel from B to A:

$U_A U_B|\psi\rangle$

GR: Dynamical causal structure



- Causal structure depends on the stress-energy tensor of the matter degrees of freedom in the causal past of the events
- The order between the events is swapped in **all** reference frames

Quantum controlled causal order

(gravitational quantum switch)

Assumptions:

1) Macroscopically distinguishable states of physical systems can be assigned orthogonal quantum states

2) Gravitational time dilation in a semiclassical limit reduces to that predicted by general relativity

3) The quantum superposition principle holds regardless of the mass of the superposed systems

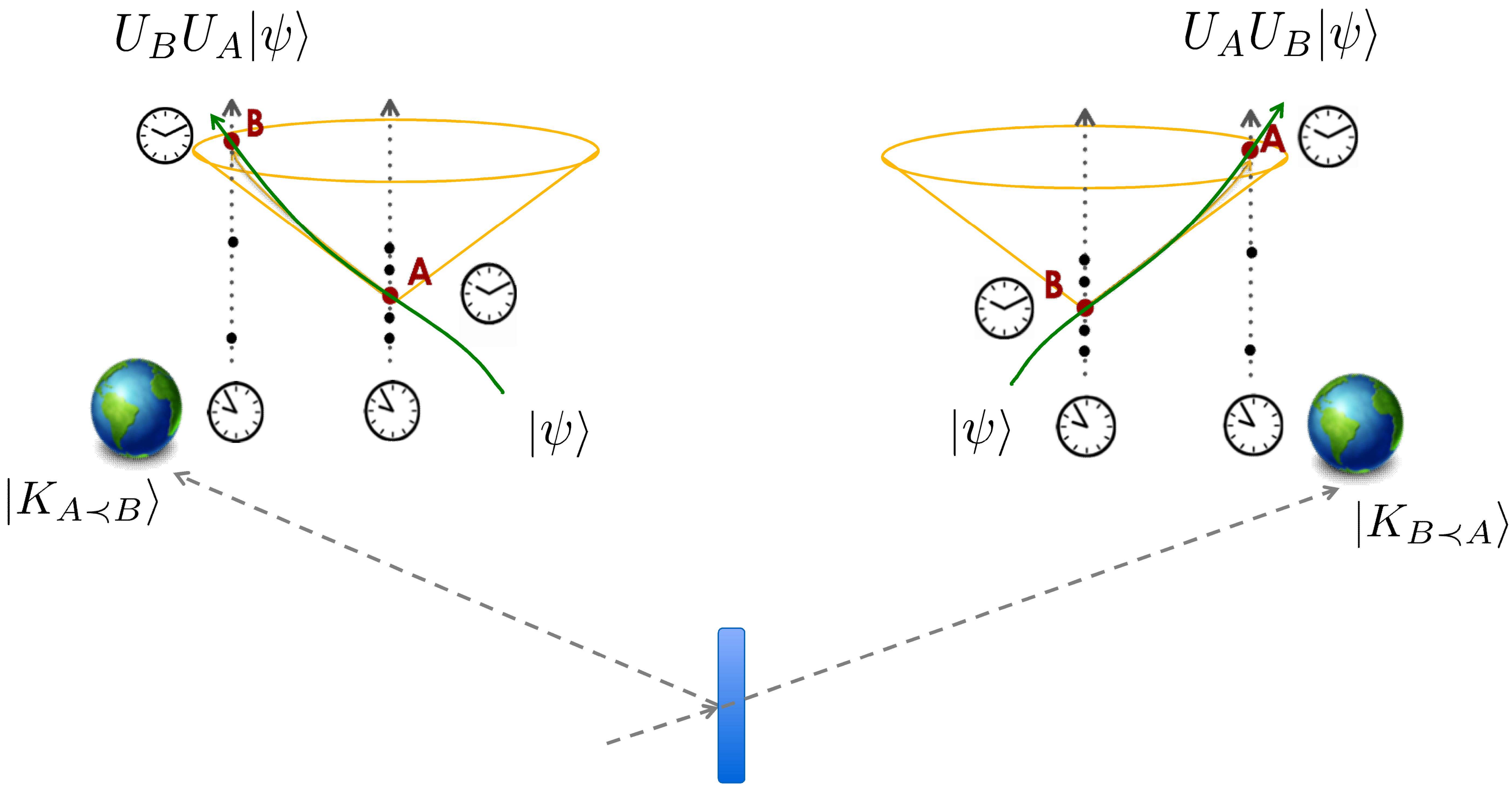
Due to 1) one can assign quantum states $|K_{A\prec B}\rangle, |K_{B\prec A}\rangle$ to the two mass configurations, s.t. $\langle K_{B\prec A}|K_{A\prec B}\rangle = 0$.

Each of the states is „semiclassical". Following 2) preparation of the states produce different causal orders.

Due to 3) the state $\frac{1}{\sqrt{2}}(|K_{A\prec B}\rangle + |K_{B\prec A}\rangle)$ is possible.

Quantum controlled causal order

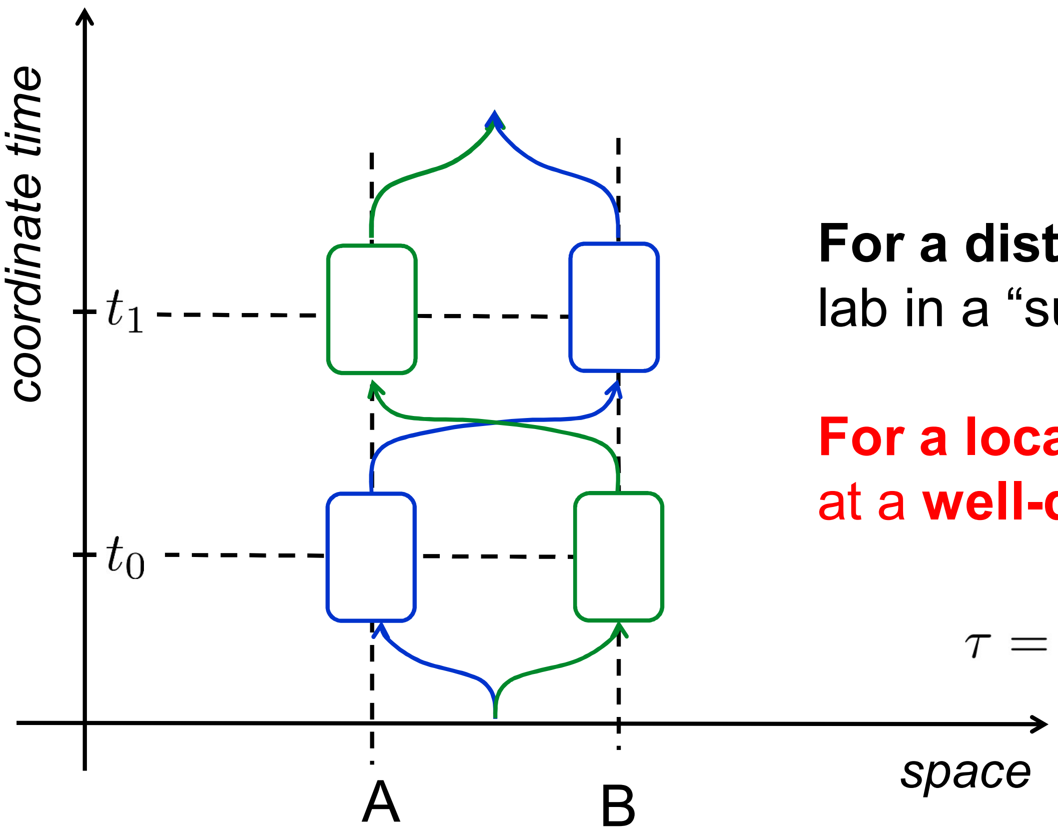
(gravitational quantum switch)



$$|W\rangle = |K_{A\prec B}\rangle|\psi\rangle^{A_1}|\mathbb{1}\rangle\rangle^{A_2B_1}|\mathbb{1}\rangle\rangle^{B_2C_1} + |K_{B\prec A}\rangle^{cnt}|\psi\rangle^{B_1}|\mathbb{1}\rangle\rangle^{B_2A_1}|\mathbb{1}\rangle\rangle^{A_2C_1}$$

M. Zych, F. Costa, I. Pikovski and Č.B., arXiv:1708.00248

The view of a distant observer



Time foliation with respect to the coordinate time (local time of a distant observer)

For a distant observer the system enters each lab in a “superposition of **two** coordinate times”.

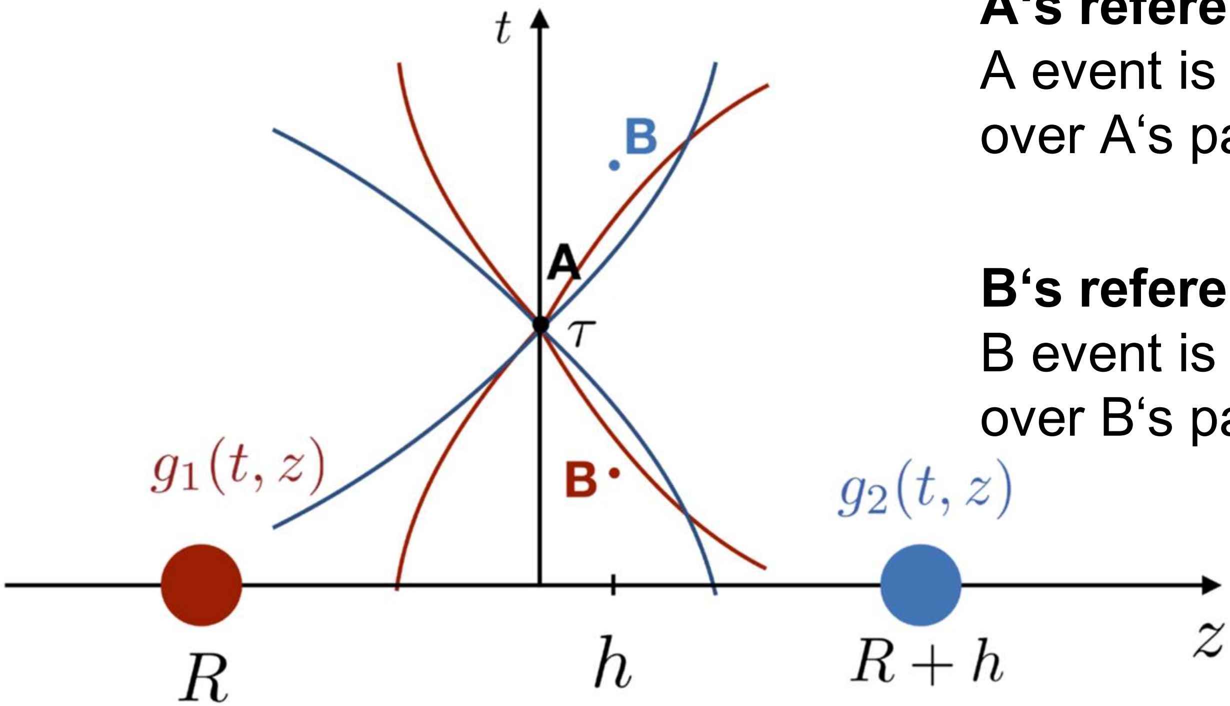
For a local observer the system enters each lab at a **well-defined** proper time.

$$\tau = \sqrt{-g_{00}(K_{A \prec B})}t_0 = \sqrt{-g_{00}(K_{B \prec A})}t_1$$

The view of a local observer

Observer-dependent localisation of event:

The gravitational quantum switch



A‘s reference frame:
A event is local, B‘s event is „spread“ over A‘s past and future

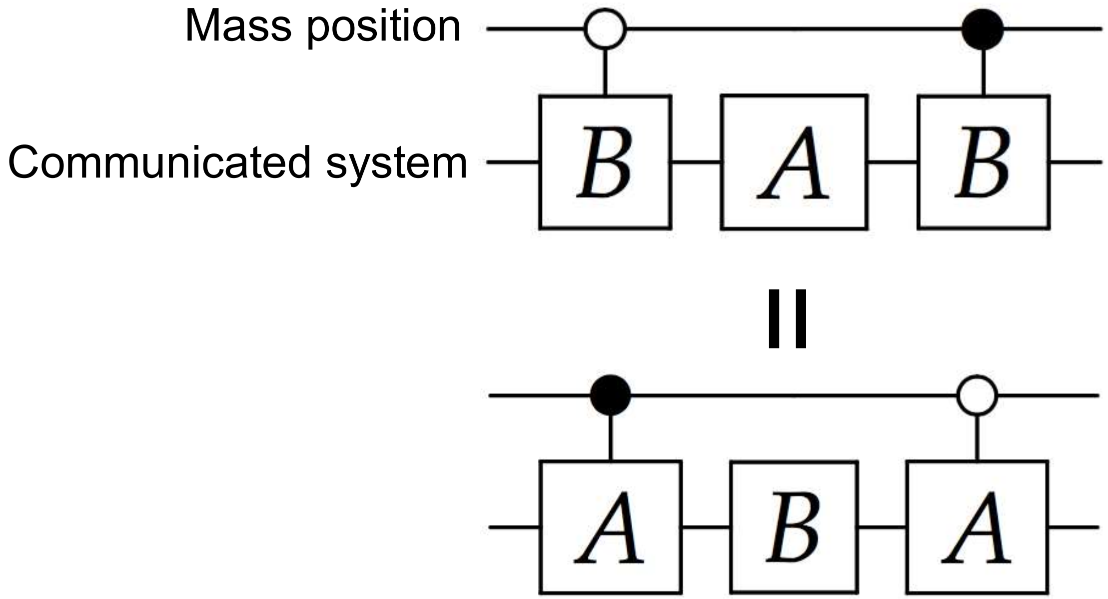
B‘s reference frame:
B event is local, A‘s event is „spread“ over B‘s past and future

P. Allard Guérin, Č.B., New J. Phys. **20**, 103031 (2018)

The view of a local observer

Observer-dependent localisation of event:

The quantum switch



P. Allard Guérin, Č.B., New J. Phys. **20**, 103031 (2018)

Summary

- Global causal order need not be a necessary element of quantum theory.
- There exist “causally non-separable processes” (the quantum switch).
- (Not shown) Linear advantage in computation and exponential reduction of communication complexity using the resource
- The quantum switch can be realized by spatial superposition of a large mass. Relation to quantum gravity theories? Effects at the Planck’s scale?
- (Not shown) There are processes that display a strong violation of causality (violate “causal inequalities”), but we do not know whether they can be realized in nature.

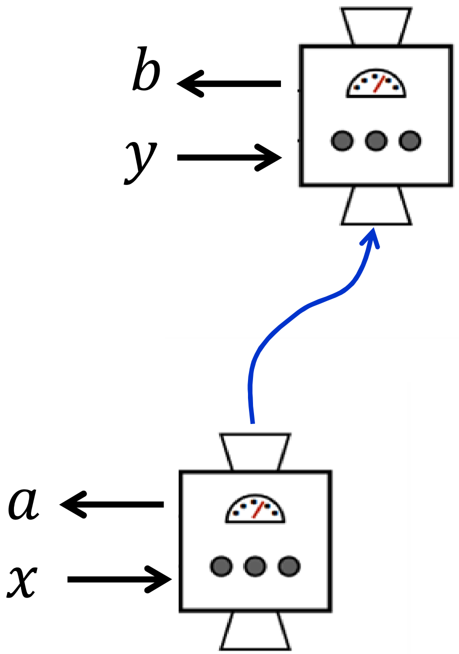
Thank you!

CoQuS Complex Quantum Systems
FWF Der Wissenschaftsfonds.
JOHN TEMPLETON FOUNDATION
SUPPORTING SCIENCE ~ INVESTING IN THE BIG QUESTIONS
TEQ

Causal Inequalities

One-directional signalling

Device-independent notion of causality



One-directional signalling
"from the past to the future"

$x,\ y$: "free variables" = **measurement settings**, statistically independent of "the rest of the experiment"

$a,\ b$: **measurement outcomes**

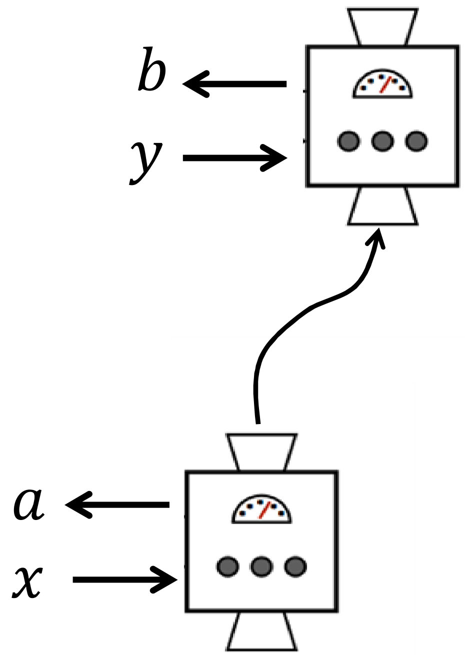
$$\sum_a p^{A \leq B}(a, b|x, y) = p^{A \leq B}(b|x, y)$$

$$\sum_b p^{A \leq B}(a, b|x, y) = p^{A \leq B}(a|x)$$

Causal inequalities

Causal correlations: either A signals to B or B signals to A, or no-signalling or a convex combination of these situations

$$p^{caus}(a,b|x,y) = \lambda p^{A\leq B}(a,b|x,y) + (1-\lambda)p^{B\leq A}(a,b|x,y)$$



Causal correlations satisfy causal inequalities, which are facets of the causal polytope.

$$p(a=y, b=x) \leq \frac{1}{2}$$

"Guess my neighbour's input game"

The switch satisfies causal inequalities.

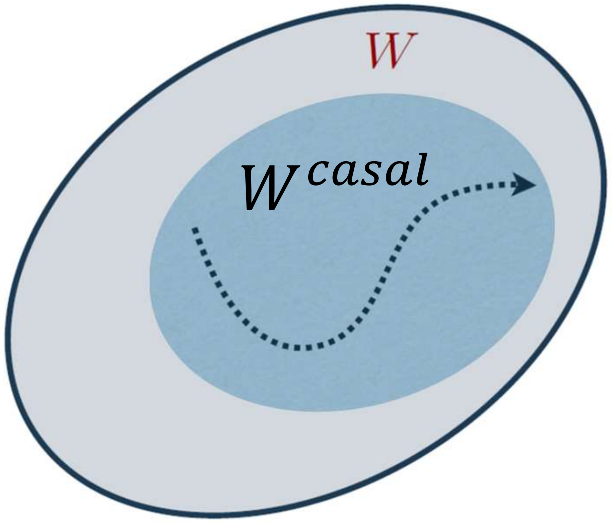
There are process matrices that **violate causal inequalities**, but we do not know if these "processes" can be realised in laboratory.

C. Branciard, M. Araújo, F. Costa, A. Feix, and Č. Brukner, New J. Phys. **18**, 013008 (2016).
A. A. Abbott, C. Giarmatzi, F. Costa, C. Branciard, Phys. Rev. A **94**, 032131 (2016)

Transformations of the processes

Can we obtain a causally nonseparable W' from a causally separable process W?

Higher-order maps: $W' = \mathcal{A}(\mathcal{W})$



All continuous and reversible process matrix transformations are local unitary operations in each party's input and output Hilbert space.

Continuous and reversible transformations always preserve the causal order

P. Perinotti. (2016) Preprint at https://arxiv.org/abs/1612.05099

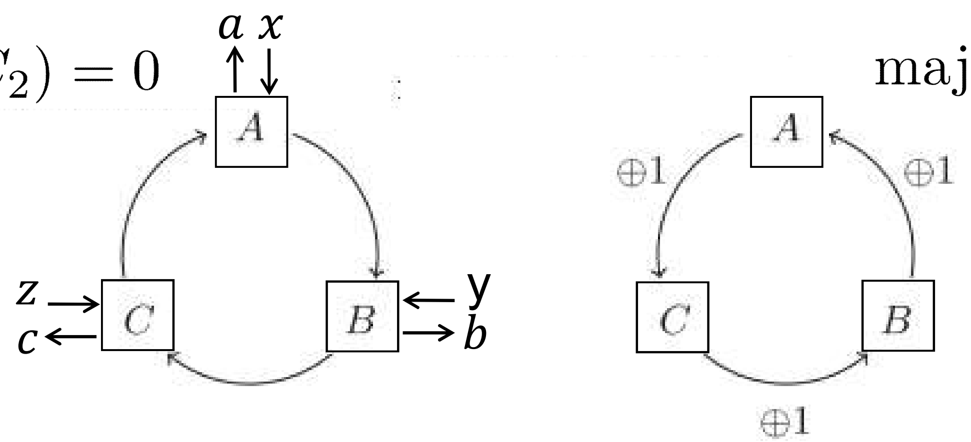
G. Chiribella, G. M. D'Ariano, and P. Perinotti. PRA (2009)

E. Castro-Ruiz, F. Giacomini, Č. B., Phys. Rev. X 8, 011047 (2018)

Violation of causal inequalities

There are process matrices that violate causal inequalities, but we do not know if these “processes” can be realised in laboratory.

$\mathrm{maj}(A_2, B_2, C_2) = 0$ $\qquad$ $\mathrm{maj}(A_2, B_2, C_2) = 1$

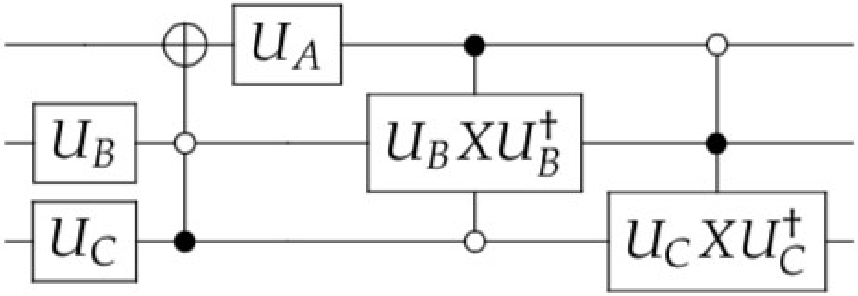


The **classical** process matrix that enables violation of a three-partite causal inequality. “Loops” with **no** “grandfather paradoxes”.

Causal inequality:

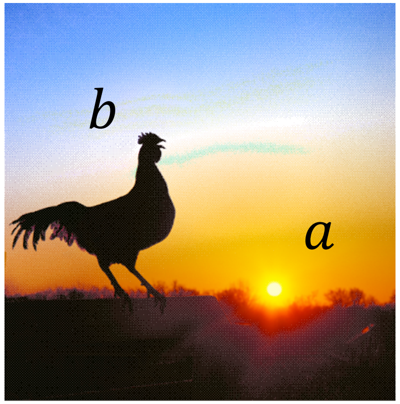
$$P_{succ} = \frac{1}{2}(P(a = z, c = y, b = x | \mathrm{maj}(x, y, z) = 0) \quad \leq \frac{3}{4} \text{ causal W}$$
$$+ P(a = \bar{y}, b = \bar{z}, c = \bar{x} | \mathrm{maj}(x, y, z) = 1) \quad = 1 \text{ the "non-causal" W}$$

Ä. Baumeler, S. Wolf, New J. Phys. **18,** 013036 (2016)



P. Allard Guérin, Č.B., Observer-dependent locality of quantum events, arXiv:1805.12429

"Correlation does not imply causation"



Need for **interventions** ("free variables") independent of the two:

a: The sun is rising or not
b: The rooster is crowing or not

x: Switching the sun on & off (hard)
y: Making a chicken soup or not

$$p(a, b)$$

$$p(a, b|x, y)$$

$$\sum_a p(a, b|x, y) = p(b|x, y)$$

$$\sum_b p(a, b|x, y) = p(a|x)$$

Conclusion: The sun will rise even if we cook the soup, but the rooster will not crow, if we switch off the sun.

The Choi-Jamilolkowski isomorphism

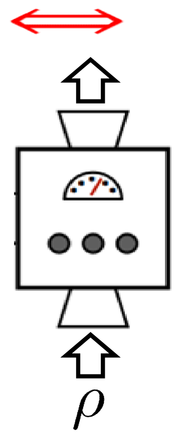
For Kraus operators:

$$K = \sum_{ij} k_{ij} |i\rangle\langle j| \;\rightarrow\; |K\rangle = \sum_{ij} k_{ji} |j\rangle|i\rangle = \sum_i K|i\rangle \otimes |i\rangle$$

For maps:

$$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2) \quad \Leftrightarrow \quad M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$$

$$M = (\mathcal{M} \otimes \mathbb{1})(|\Phi^+\rangle\langle\Phi^+|) = \sum_{ij} |i\rangle\langle j|^1 \otimes \mathcal{M}(|i\rangle\langle j|)^2$$



Inverse isomorphism: $\mathcal{M}(\rho) = \mathrm{Tr}_1[M(\rho^T \otimes \mathbb{1})]$