Sigma–Models & Non–geometric Fluxes in String and M theories

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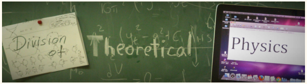

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Hrvatska zaklada za znanost

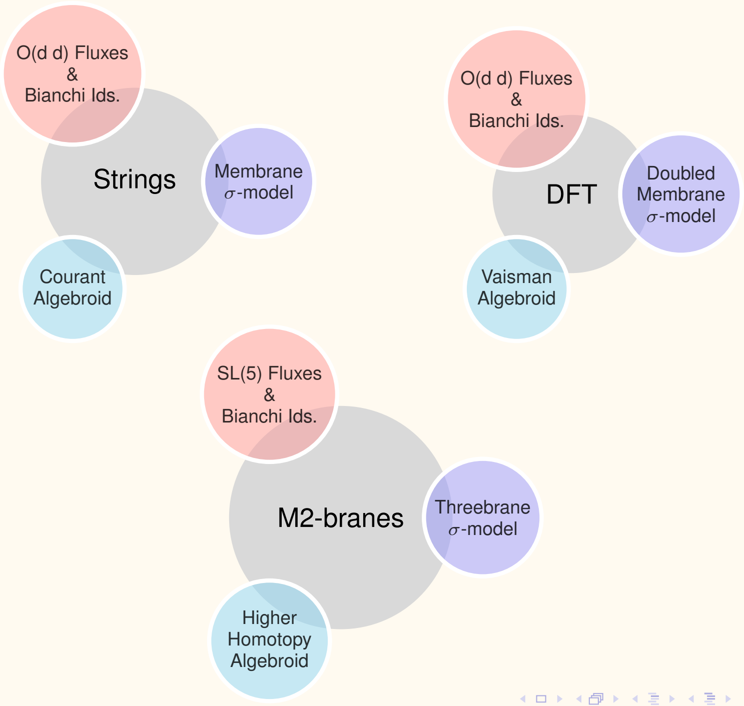
Based on:

1901.07775 with **L. Jonke** - **D. Lüst** - **R. J. Szabo**

1802.07003 with **L. Jonke** - **F. S. Khoo** - **R. J. Szabo**

1311.4878 & 1505.05457 with **L. Jonke** - **O. Lechtenfeld**

Quantum Spacetime ’19


O(d d) Fluxes
&
Bianchi Ids.
Strings
Membrane
σ-model
Courant
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O(d d) Fluxes
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DFT
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SL(5) Fluxes
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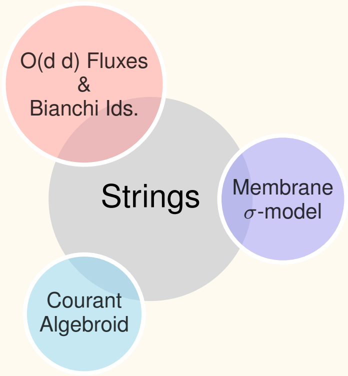
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Three equations, three viewpoints

$$\eta^{IJ}\,\rho^i{}_I\,\rho^j{}_J = 0\,, \quad i = 1,\ldots,d$$

$$\rho^i{}_I\,\partial_i\rho^j{}_J - \rho^i{}_J\,\partial_i\rho^j{}_I - \eta^{KL}\,\rho^j{}_K\,T_{LIJ} = 0\,, \quad I = 1,\ldots 2d$$

$$4\,\rho^i{}_{[L}\,\partial_i T_{IJK]} + 3\,\eta^{MN}\,T_{M[IJ}\,T_{KL]N} = 0\,.$$

η is the $O(d,d)$-invariant metric: $(\eta_{IJ}) = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$

Viewpoint 1: Flux compactifications of strings

ρ: Potential for geometric and non-geometric fluxes $\rightsquigarrow \rho^i{}_I = (e^i{}_a, e^i{}_b \beta^{ba})$

T: The (NSNS) fluxes $\rightsquigarrow T_{IJK} = (H_{abc}, f_{ab}{}^c, Q_a{}^{bc}, R^{abc})$, typically related by T-duality

$$H_{abc} \overset{T_c}{\longleftrightarrow} f_{ab}{}^c \overset{T_b}{\longleftrightarrow} Q_a{}^{bc} \overset{T_a}{\longleftrightarrow} R^{abc} \; .$$

Decomposing the second equation, one obtains the local expressions for all fluxes:

$$H_{abc} = 3\, \nabla_{[a} B_{bc]} \, ,$$

$$F_{ab}{}^c = 2\, e^c{}_j \, e_{[a}{}^i \, \partial_i e_{b]}{}^j + \beta^{cd} \, H_{abd} \; ,$$

$$Q_a{}^{bc} = \partial_a \beta^{bc} + \beta^{bd} \, f_{ad}{}^c - \beta^{cd} \, f_{ad}{}^b + \beta^{bd} \, \beta^{ce} \, H_{ade} \; ,$$

$$R^{abc} = 3\, \beta^{[a\underline{d}} \, \nabla_d \beta^{bc]} + \beta^{ad} \, \beta^{be} \, \beta^{cf} \, H_{def} \; ,$$

Decomposing the third equation, one obtains their Bianchi identities.

Viewpoint 2: Gauge structure of membrane sigma models

Courant sigma-model for maps $X = (X^i) : \Sigma_3 \to M_d$ with Wess-Zumino term T

Hofman, Park '02; Ikeda '02

$$S_0[X, A, F] = \int_{\Sigma_3} \left(F_i \wedge \mathrm{d}X^i + \tfrac{1}{2}\, \eta_{IJ}\, A^I \wedge \mathrm{d}A^J - \rho^i{}_I(X)\, A^I \wedge F_i + \tfrac{1}{6}\, T_{IJK}(X)\, A^I \wedge A^J \wedge A^K \right) .$$

The 1-forms A take values in the (pull-back) bundle $E = TM \oplus T^*M$.

The 3 equations encode the gauge symmetries of the theory & their on-shell closure

$$\begin{aligned}
\delta_\epsilon X^i &= \rho^i_J \epsilon^J \ , \\
\delta_\epsilon A^I &= \mathrm{d}\epsilon^I + \eta^{IN} T_{NJK} A^J \epsilon^K - \eta^{IJ} \rho^i{}_J t_i \ , \\
\delta_\epsilon F_m &= -\mathrm{d}t_m - \partial_m \rho^j{}_J A^J t_j - \epsilon^J \partial_m \rho^i{}_J F_i + \tfrac{1}{2} \epsilon^J \partial_m T_{ILJ} A^I \wedge A^L \ .
\end{aligned}$$

Equivalently, the classical master equation of the corresponding BV action is satisfied.

On-shell (w/ suitable boundary kinetic term) $\rightsquigarrow$ strings in (non-)geometric backgrounds.

Mylonas, Schupp, Szabo '12; ACh, Jonke, Lechtenfeld '15

Viewpoint 3: Axioms of a Courant algebroid

Axiomatic organisation of the properties of the (twisted) Courant bracket on $TM \oplus T^*M$

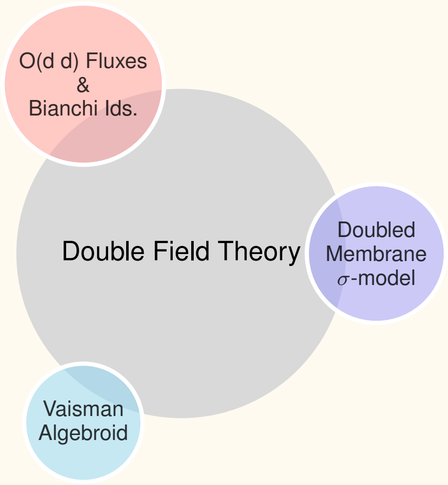
Courant '90; Liu, Weinstein, Xu '95; Ševera

$$[X+\xi, Y+\eta] \quad = \quad [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \tfrac{1}{2}\,\mathrm{d}(\iota_X\eta - \iota_Y\xi) + H(X,Y)\ .$$

$(E \xrightarrow{\pi} M, [\cdot,\cdot], \langle\cdot,\cdot\rangle, \rho : E \to TM)$, such that for $A, B, C \in \Gamma(E)$ and $f, g \in C^\infty(M)$:

1. $[[A,B],C] + \text{c.p.} = \mathcal{D}\mathcal{N}(A,B,C)\ , \quad \text{where} \quad \mathcal{N}(A,B,C) = \tfrac{1}{3}\langle [A,B], C\rangle + \text{c.p.}\ ,$
2. $[A, fB] = f[A,B] + (\rho(A)f)B - \langle A,B\rangle \mathcal{D}f\ ,$
3. $\rho(C)\langle A,B\rangle = \langle [C,A] + \mathcal{D}\langle C,A\rangle, B\rangle + \langle [C,B] + \mathcal{D}\langle C,B\rangle, A\rangle\ ,$
4. $\rho[A,B] = [\rho(A), \rho(B)]\ ,$
5. $\rho\circ\mathcal{D} = 0 \quad \Leftrightarrow \quad \langle \mathcal{D}f, \mathcal{D}g\rangle = 0\ .$

In local coordinates, they become the three equations...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DFT: Three equations, three viewpoints

$$\eta^{IJ}\rho^{K}{}_{I}\rho^{L}{}_{J} = \eta^{KL}$$

$$2\rho^{L}{}_{[I}\partial_{\underline{L}}\rho^{K}{}_{J]} - \eta^{LM}\rho^{K}{}_{L}\hat{T}_{MIJ} = \rho_{L[I}\partial^{K}\rho^{L}{}_{J]}$$

$$4\rho^{M}{}_{[L}\partial_{\underline{M}}\hat{T}_{IJK]} + 3\eta^{MN}\hat{T}_{M[IJ}\hat{T}_{KL]N} = \mathcal{Z}_{IJKL}\ .$$

Viewpoint 1: Duality-symmetric formulation of fluxes

ρ and $\hat{T}$ become a generalized vielbein and the T-duality-symmetric DFT fluxes.

Decomposing the second equation, one obtains the augmented local expressions:

Aldazabal, Baron, Marques, Nunez '11; Geissbühler '11

$$
\begin{aligned}
H_{ijk} &= 3\,\partial_{[i} B_{jk]} + 3\,B_{[i\underline{l}}\,\tilde{\partial}^l B_{jk]} \;, \\
f_{ij}{}^k &= \tilde{\partial}^k B_{ij} + \beta^{kl}\,H_{lij} \;, \\
Q_k{}^{ij} &= \partial_k \beta^{ij} + B_{kl}\,\tilde{\partial}^l \beta^{ij} + 2\,\beta^{l[i}\,\tilde{\partial}^{j]} B_{lk} + \beta^{il}\,\beta^{jm}\,H_{lmk} \;, \\
R^{ijk} &= 3\,\tilde{\partial}^{[i} \beta^{jk]} + 3\,\beta^{[i\underline{l}}\,\partial_l \beta^{jk]} \\
&\quad + 3\,B_{lm}\,\beta^{[i\underline{l}}\,\tilde{\partial}^m \beta^{jk]} + 3\,\beta^{[i\underline{l}}\,\beta^{j\underline{m}}\,\tilde{\partial}^{k]} B_{lm} + \beta^{il}\,\beta^{jm}\,\beta^{kn}\,H_{lmn} \;,
\end{aligned}
$$

Decomposing the third equation, one obtains their corresponding Bianchi identities.

Viewpoint 2: Doubled membrane sigma-model

A double worth of scalar fields $(\mathbb{X}^I) = (X^i, \widetilde{X}_i)$: ACh, Jonke, Lechtenfeld '15; ACh, Jonke, Khoo, Szabo '18

$$S[\mathbb{X}, A, F] = \int \Big(F_I \wedge \mathrm{d}\mathbb{X}^I + \eta_{IJ}\, A^I \wedge \mathrm{d}A^J - (\rho_+)^I{}_J\, A^J \wedge F_I + \tfrac{1}{3}\, \widehat{T}_{IJK}\, A^I \wedge A^J \wedge A^K \Big) \ .$$

Gauge symmetries and their closure (or the classical master equation), obstructed.

Obstructions vanish when a strong constraint is satisfied: $\eta^{IJ}\partial_I \otimes \partial_J = 0$. cf. Hull, Zwiebach '09

On-shell, describes non-commutative/non-associative string flux backgrounds.

Viewpoint 3: (Pre-)DFT (Vaisman) Algebroid

Axiomatic organisation of the properties of the C-bracket: Siegel '93; Hull, Zwiebach '09

$$[\![A, B]\!]^J = A^K\, \partial_K B^J - \tfrac{1}{2}\, A^K\, \partial^J B_K - \{A \leftrightarrow B\}\ .$$

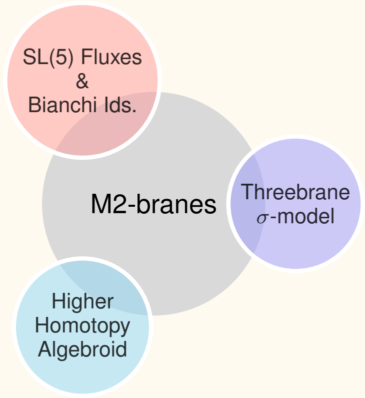
$(L_+ \xrightarrow{\pi} \mathcal{M}, [\![\cdot,\cdot]\!], \langle\cdot,\cdot\rangle_{L_+}, \rho_+ : L_+ \to T\mathcal{M})$ satisfying

2. $[\![A, fB]\!] = f[\![A, B]\!] + (\rho_+(A)f)\, B - \langle A, B\rangle_{L_+} \mathcal{D}_+ f\ ,$
3. $\langle [\![C, A]\!] + \mathcal{D}_+\langle C, A\rangle_{L_+}, B\rangle_{L_+} + \langle [\![C, B]\!] + \mathcal{D}_+\langle C, B\rangle_{L_+}, A\rangle_{L_+} = \rho_+(C)\langle A, B\rangle_{L_+}\ ,$
5. $\langle \mathcal{D}_+ f, \mathcal{D}_+ g\rangle_{L_+} = \tfrac{1}{4}\, \langle \mathrm{d}f, \mathrm{d}g\rangle_{L_+}\ .$

Notably, the modified Jacobi, homomorphism and kernel properties are obstructed.

In general, by relaxing properties one obtains a host of intermediate structures
cf. Vaisman '04; Hansen, Strobl '09; Vaisman '12; Bruce, Grabowski '16

$$\text{Pre-DFT algebroid} \xleftarrow{\ \not{5}\ } \text{Ante-Courant algebroid} \xleftarrow{\ \not{4}\ } \text{Pre-Courant algebroid} \xleftarrow{\ \not{1}\ } \text{Courant algebroid}$$


SL(5) Fluxes
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Higher
Homotopy
Algebroid

Three + Two equations, three viewpoints

$$\rho^i{}_I \, S^{IJ} = 0 \; ,$$

$$\rho^i{}_I \, \partial_i S^{JK} + S^{LJ} \, T^K{}_{IL} + S^{LK} \, T^J{}_{IL} = 0 \; ,$$

$$\rho^i{}_I \, \partial_i \rho^j{}_J - \rho^i{}_J \, \partial_i \rho^j{}_I - \rho^i{}_K \, T^K{}_{IJ} = 0 \; ,$$

$$3 \rho^i{}_{[I} \, \partial_i T^J{}_{KL]} + S^{JM} \, G_{KLIM} - 3 T^J{}_{M[K} \, T^M{}_{LI]} = 0 \; ,$$

$$\rho^i{}_{[I} \, \partial_i G_{JKLM]} + T^N{}_{[IJ} \, G_{KLM]N} = 0 \; ,$$

N.B. Index $I = 1, \ldots, \dim E$, where E is a vector bundle.

Viewpoint 1: Non-geometric M theory fluxes

When 11D supergravity is compactified on T^4 $\rightsquigarrow$ SL(5) (U-duality) symmetry.

Cremmer, Julia '78, '81

Fluxes are irreps of SL(5): G_{abcd} , $F_{ab}{}^{c}$, $Q_{a}{}^{bcd}$, R^{abcde}.

Blair, Malek '14

Alternatively, twists of the (higher) Courant bracket on $TM \oplus \wedge^2 T^*M$ for M being 4D.

Their general local coordinate expressions are found to be

$$G_{abcd} = 4\,\nabla_{[a} C_{bcd]}\ ,$$

$$F_{ab}{}^{c} = f_{ab}{}^{c} - \tfrac{1}{2}\, G_{abde}\, \Omega^{dec}\ ,$$

$$Q_{a}{}^{bcd} = \tfrac{1}{2}\left(\partial_a \Omega^{bcd} + 3\,\Omega^{e[bc}\, f_{ae}{}^{d]} - \tfrac{1}{2}\,\Omega^{def}\,\delta_a^{[b}\, f_{ef}{}^{c]} - \tfrac{1}{2}\,\Omega^{e[bc}\, G_{aefg}\,\Omega^{d]fg}\right)\ ,$$

$$R^{ab,cd,e} = 2\,\widehat{\nabla}^{a[b}\,\Omega^{cde]} - 2\,\widehat{\nabla}^{b[a}\,\Omega^{cde]} - 2\,\widehat{\nabla}^{c[d}\,\Omega^{abe]} + 2\,\widehat{\nabla}^{d[c}\,\Omega^{abe]}\ ,$$

where $\widehat{\nabla}^{ab} = \frac{1}{4}\,\Omega^{abc}\,\nabla_c$ and $f_{ab}{}^{c} = 2\, e^{c}{}_{j}\, e_{[a}{}^{i}\,\partial_i e_{b]}{}^{j} =: 2\,\Gamma_{[ab]}{}^{c}$ the purely geometric flux.

These and their BIs can actually be directly obtained from the three equations ($S = 0$...)

Viewpoints 2 & 3: Threebrane sigma-model & higher homotopy algebroids

The starting point is is a topological threebrane sigma-model with action functional
Ikeda, Uchino '10; Kökenyesi, Sinkovics, Szabo '18

$$S[X,\alpha,A,F]=\int_{\Sigma_4}\Big(F_i\wedge \mathrm{d}X^i-\alpha_I\wedge\mathrm{d}A^I+\rho^i{}_I(X)\,F_i\wedge A^I+\tfrac{1}{2}\,S^{IJ}(X)\,\alpha_I\wedge\alpha_J$$
$$+\tfrac{1}{2}\,T^I{}_{JK}(X)\,\alpha_I\wedge A^J\wedge A^K+\tfrac{1}{4!}\,G_{IJKL}(X)\,A^I\wedge A^J\wedge A^K\wedge A^L\Big)\ .$$

Here, $A\in\Omega^1(\Sigma_4,X^*E)$ & $\alpha\in\Omega^2(\Sigma_4,X^*E^*)$. Gauge structure $\rightsquigarrow$ the five equations.

(Moreover, local form of the axioms for a structure called Lie algebroid up to homotopy.)
Ikeda, Uchino '10; Grützmann '10

Specialize: $E=TM\oplus\bigwedge^2T^*M$ (N.B. Not a standard choice!), then $T=\mathrm{SL}(5)$ fluxes.

Hierarchy of structures

dim Σ	σ-model	Scalars	1-forms	2-forms	3-forms
2	Poisson	X^i	$F_i \in \Gamma(X^* T^* M)$	—	—
3	Courant	X^i	$A^I \in \Gamma(X^* E)$	$F_i \in \Gamma(X^* T^* M)$	—
4	Threebrane	X^i	$A^I \in \Gamma(X^* E)$	$\alpha_I \in \Gamma(X^* E^*)$	$F_i \in \Gamma(X^* T^* M)$

dim Σ	σ-model	Algebroid	Quantization
2	Poisson	Lie	Open string non-commutativity Kontsevich '97; Cattaneo, Felder '99
3	Courant	Courant	Closed string non-associativity Mylonas, Schupp, Szabo '12
4	Threebrane	Homotopy	...