Random Fuzzy Spaces in the Spectral Triple formalism

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Quantum Spacetime '19, Bratislava, Slovakia

Outline

- Motivation and definitions
- Preliminary studies in a symmetry-breaking potential
- The (2,0) geometry as a toy model
- Higher geometries

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Geometry is encoded in spectral data

$$(A, H, D; J, \gamma)$$

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The particle content of the Standard Model is described by the following data

$$(A_F, H_F, D_F; J_F, \gamma_F)$$

$$A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

$$H_F = \mathbb{C}^{96}$$

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To get Einstein-Hilbert plus the Standard Model one considers the spectral triple obtained by tensoring the commutative manifold with the internal non-commutative finite space

$$(C^\infty(M) \otimes A_F, \;\; L^2(S) \otimes H_F, \;\; D_M \otimes 1 + \gamma_5 \otimes D_F; \;\; J_M \otimes J_F, \;\; \gamma_M \otimes \gamma_F)$$

bosonic action $\mathrm{Tr}\left(f\left(\frac{D}{\Lambda}\right)\right) \sim \int \mathcal{L}_M + \mathcal{L}_{g.f.} + \mathcal{L}_h$

fermionic action $\langle J\psi, D\psi \rangle$

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The idea: replace the commutative manifold with a fuzzy space

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Fuzzy spaces as spectral triples:

$$\mathcal{A} = M_n(\mathbb{C})$$

(arXiv:1502.05383)
Barrett

$$\mathcal{H} = \mathbb{C}^c \otimes M_n(\mathbb{C})$$

$$D = \sum_i \alpha_i \otimes \{H_i, \cdot\} + \sum_j \tau_j \otimes [L_i, \cdot]$$

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Path integration over geometries is then implemented by integration over the space of Dirac operators

$$\langle f(D)\rangle = \int f(D)e^{-S[D]}dD$$

The simplest non-trivial choice for an action

$$S = g_2 \mathrm{Tr} D^2 + \mathrm{Tr} D^4$$

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Issues:

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- Huge integral

$$D = \sum_i \alpha_i \otimes \{H_i, \cdot\} + \sum_j \tau_j \otimes [L_i, \cdot]$$

free parameters

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(arXiv:1902.03590)
Barrett, Druce, Glaser

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- A priori no guarantee of interesting behaviour or emergent geometry

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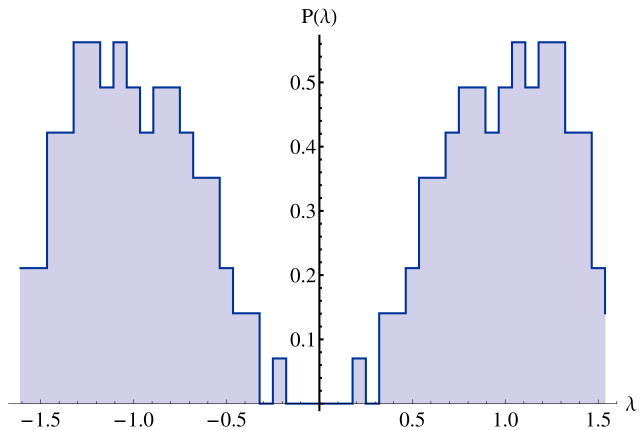
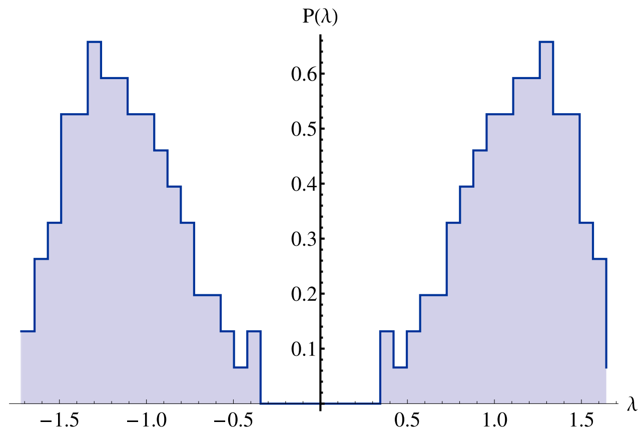
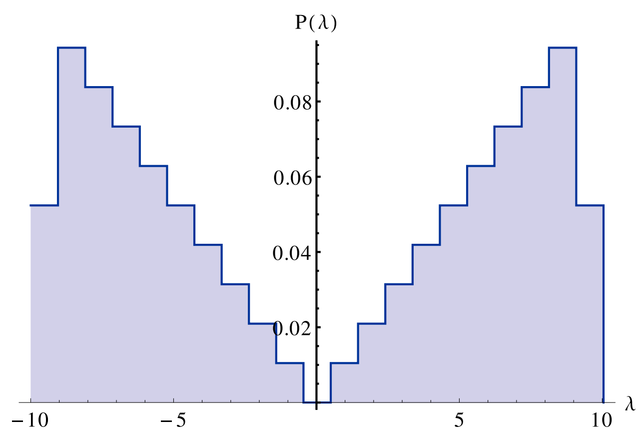
First indication of emergent geometry



(a) Type $(1,1)$, $g_2 = -2.5$



(b) Type $(2,0)$, $g_2 = -3$



(c) Type (1,3) Fuzzy S^2

(arXiv:1510.01377)
Barrett, Glaser

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The (2,0) geometry

$$D_{(2,0)} = \gamma_1 \otimes \{H_1, \cdot\} + \gamma_2 \otimes \{H_2, \cdot\}$$

$$\gamma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \gamma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

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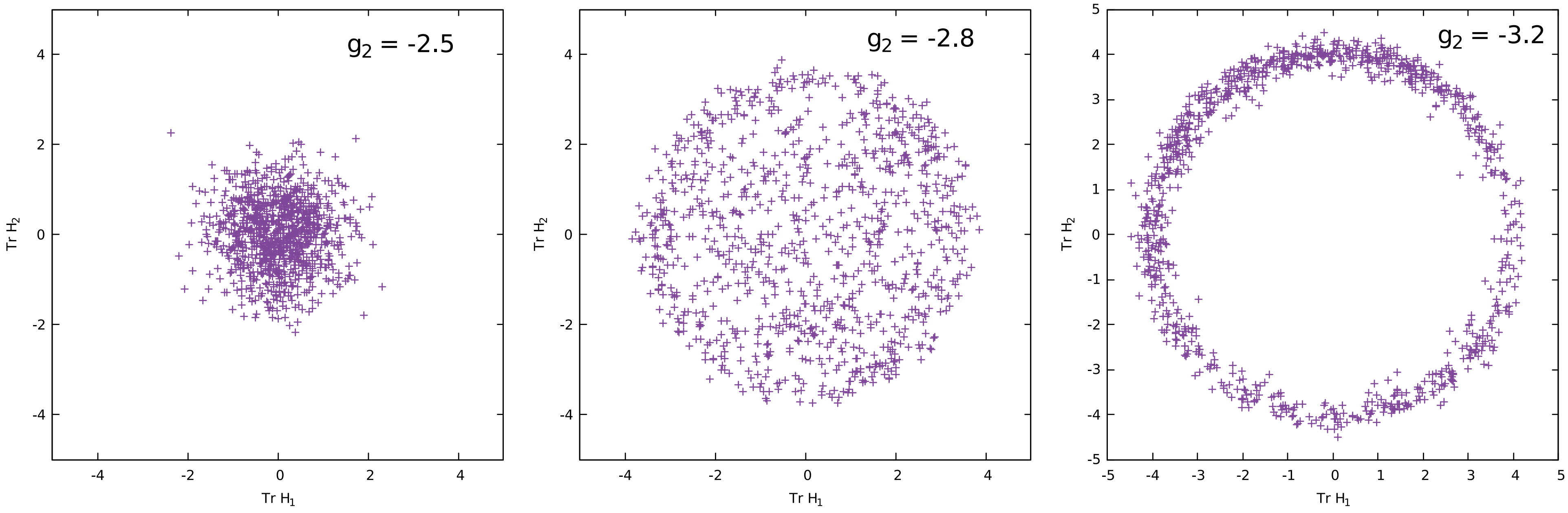
The (2,0) geometry



This suggests that the radius of the circle is an interesting observable to compute

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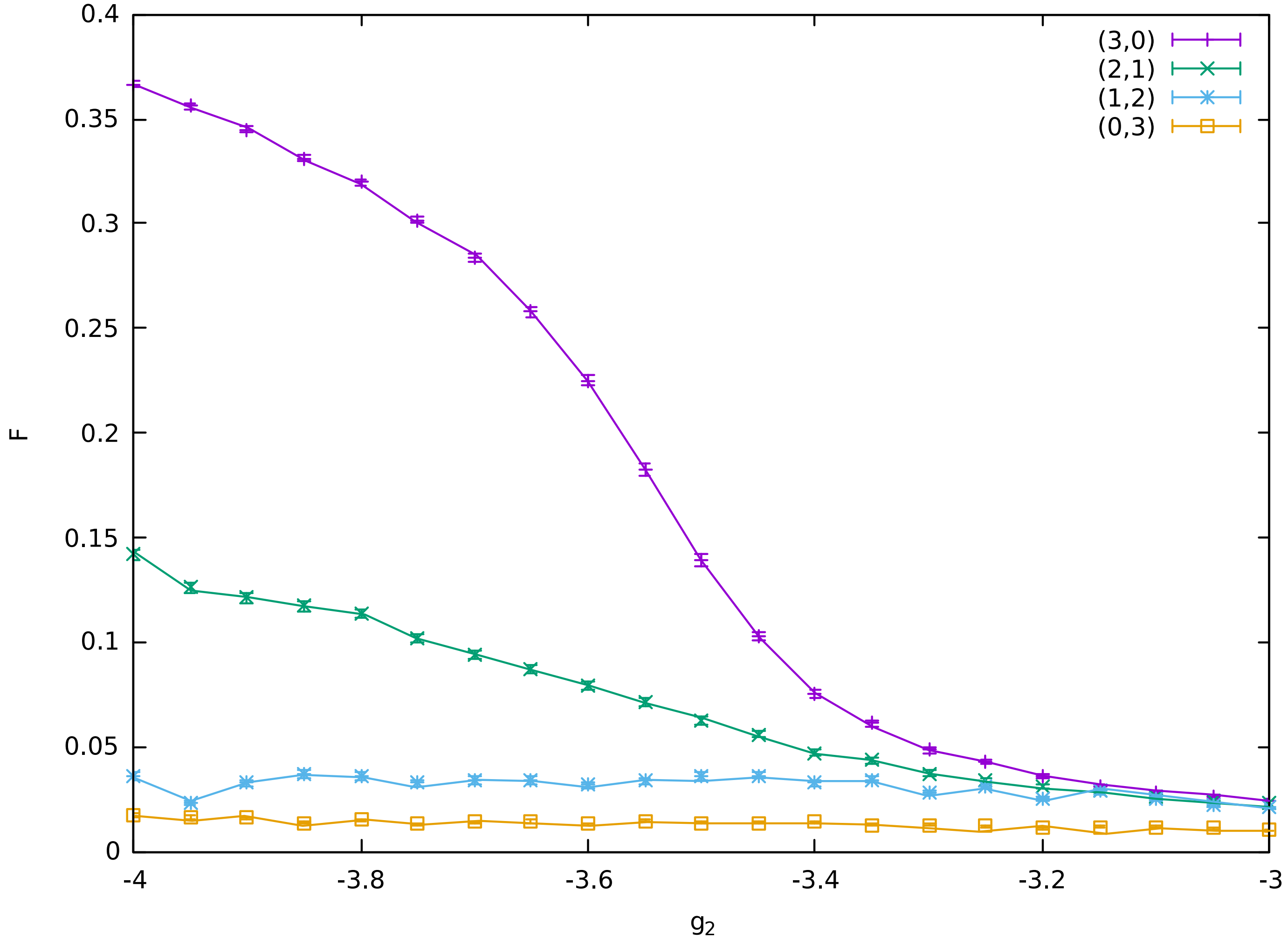
Idea for an observable to compute:

$$F = \frac{1}{\mathcal{N}} \sum_i (\mathrm{Tr}\, H_i)^2$$



p+q = 3

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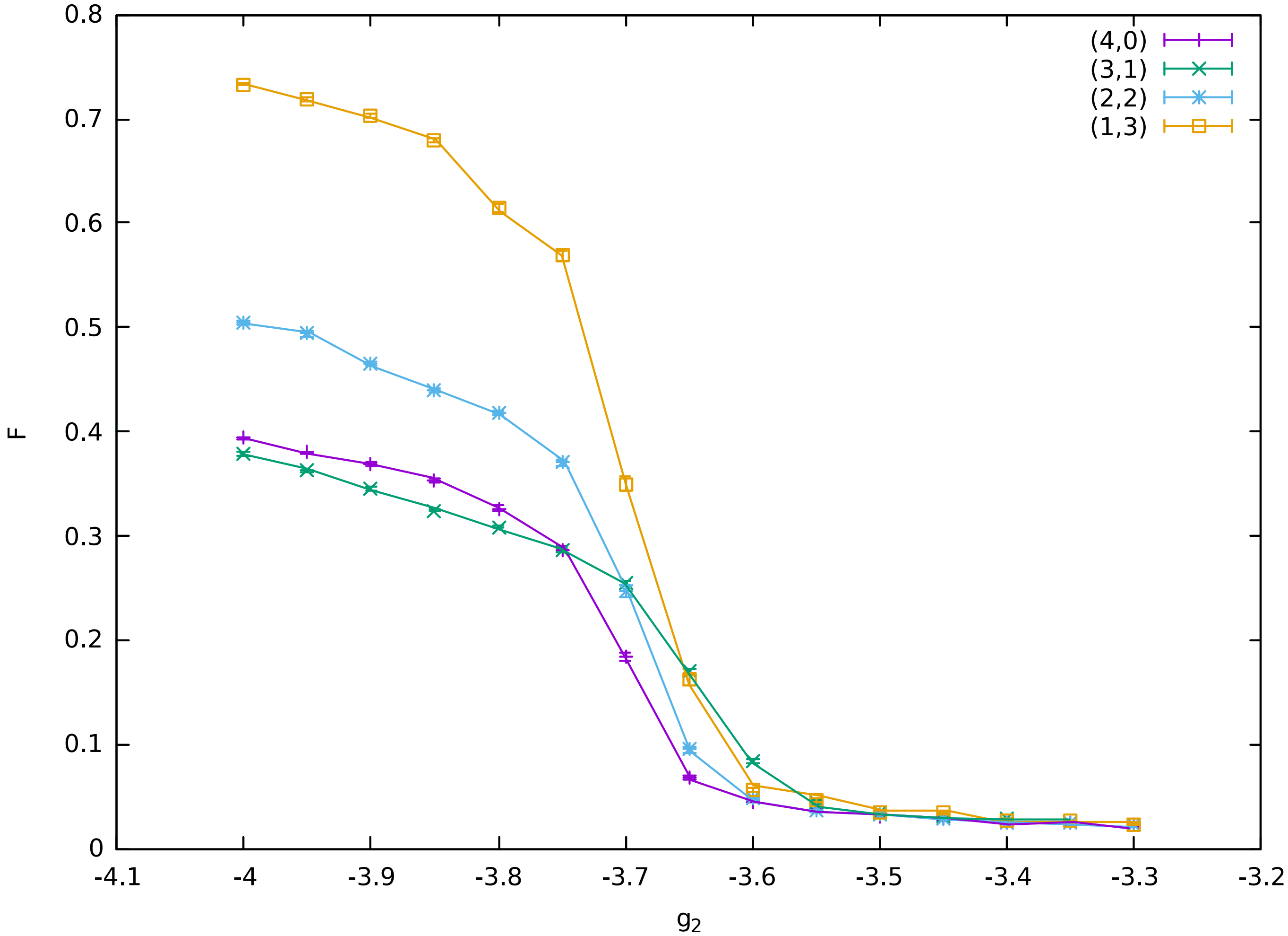

p+q = 4

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Fuzzy sphere Dirac operator

$$D_{fs} = \gamma^0 \otimes I + \sum_{j<k=1}^{3} \gamma^0 \gamma^j \gamma^k \otimes [L_{jk}, \cdot]$$

Standard generators of su(2)

(1, 3) Dirac operator

$$D_{13} = \gamma^0 \otimes \{H_0, \cdot\} + \gamma^1 \gamma^2 \gamma^3 \otimes \{H_{123}, \cdot\} +$$

$$\sum_{i=1}^{3} \gamma^i \otimes [L_i, \cdot] + \sum_{j<k=1}^{3} \gamma^0 \gamma^j \gamma^k \otimes [L_{jk}, \cdot]$$

In general they generate a much bigger algebra

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Using the Frobenius inner product

$$\langle A, B \rangle = \mathrm{Tr} A^\dagger B = ||A|| \cdot ||B|| \cos\theta$$

we can test whether the algebra of the L matrices shrinks, for example:

$$[L_{jk}, L_{lm}] \propto L_{rs}$$

by measuring the angle between them

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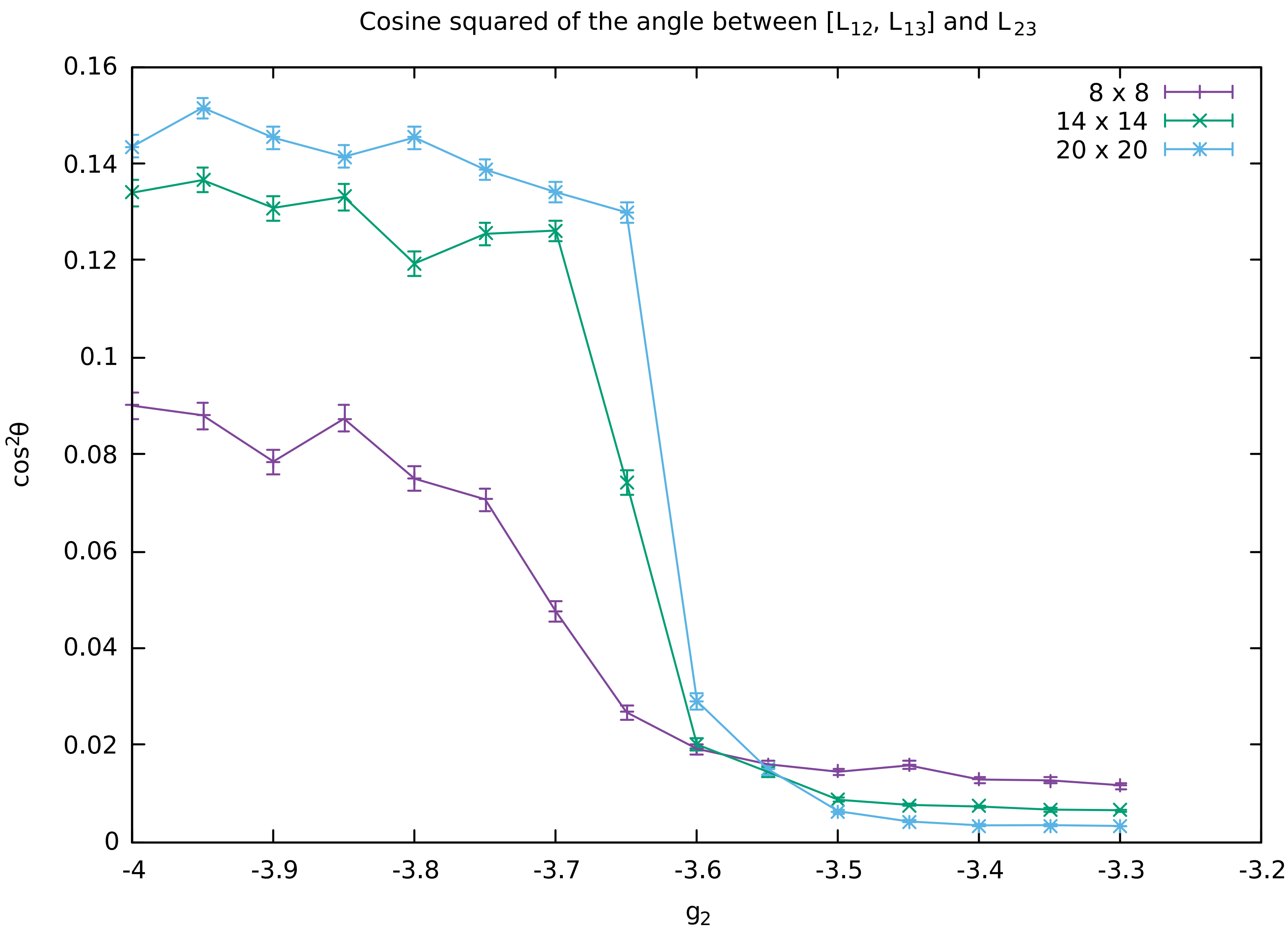

Conclusions

- Random fuzzy spaces exhibit phase transitions
- Interesting behaviour potentially emerges at the critical point
- No geometric input

To do

- Improve numerics
- Analytical understanding
- Higher powers in the action
- Lorentzian version?

Thank you for listening