Four-dimensional gravity on a covariant noncommutative space

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Einstein 4d gravity as a gauge theory

The algebra

- Employ the vielbein formulation of General Relativity
- Gauge theory of the Poincare group, ISO(1,3)
- Ten generators (Translations, P_a & LT M_{ab})

see for details: Utiyama '56, Kibble '61, McDowell-Mansuri '77, Kibble - Stelle '85

Generators satisfy the commutation relations:

$$[M_{ab}, M_{cd}] = \eta_{ac} M_{db} - \eta_{bc} M_{da} - \eta_{ad} M_{cb} + \eta_{bd} M_{ca}$$
$$[P_a, M_{bc}] = \eta_{ab} P_c - \eta_{ac} P_b \,, \qquad [P_a, P_b] = 0$$

where $\eta_{ab} = \mathrm{diag}(-1, +1, +1, +1)$ and $a, b, c, d = 1, \ldots, 4$.

The gauging procedure

- Intro of a gauge field for each generator: ${e_\mu}^a, {\omega_\mu}^{ab}$ (transl, LT)
- Define the covariant derivative → the $\mathfrak{iso}(1,3)$-valued 1-form gauge connection is:

$$A_\mu = {e_\mu}^a(x) P_a + \tfrac{1}{2} {\omega_\mu}^{ab}(x) M_{ab}$$

- Transforms in the adjoint rep, according to the rule:

$$\delta A_\mu = \partial_\mu \epsilon + [A_\mu, \epsilon]$$

- The gauge transformation parameter, $\epsilon(x)$ is expanded as:

$$\epsilon(x) = \xi^a(x) P_a + \tfrac{1}{2} \lambda^{ab}(x) M_{ab}$$

- *Combining* the above → transformations of the fields:

$$\delta {e_\mu}^a = \partial_\mu \xi^a - {e_\mu}^b {\lambda^a}_b + {\omega_\mu}^{ab} \xi_b$$
$$\delta {\omega_\mu}^{ab} = \partial_\mu \lambda^{ab} - {\lambda^a}_c {\omega_\mu}^{cb} + {\lambda^b}_c {\omega_\mu}^{ca}$$

Curvatures and action

- Curvatures of the fields are given by:

$$R_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

- Tensor $\mathrm{R}_{\mu\nu}$ is also valued in $\mathfrak{iso}(1,3)$:

$$R_{\mu\nu}(A) = T_{\mu\nu}{}^{a} P_a + R_{\mu\nu}{}^{ab} M_{ab}$$

- *Combining* the above $\rightarrow$ component tensor curvatures:

$$T_{\mu\nu}{}^{a} = \partial_\mu e_\nu{}^{a} - \partial_\nu e_\mu{}^{a} + e_\mu{}^{b}\omega_{\nu b}{}^{a} - e_\nu{}^{b}\omega_{\mu b}{}^{a}$$
$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} - \omega_\mu{}^{cb}\omega_\nu{}^{a}{}_{c} + \omega_\mu{}^{ac}\omega_{\nu c}{}^{b}$$

Stelle-West '80, Kibble - Stelle '85

- In order to obtain E-H action one has to begin with the de Sitter group, consider the Pontryagin index + use an auxiliary field, gauge fix it which breaks the symmetry to the Lorentz subgroup$\rightarrow$ scalar curvature lagrangian + torsionless condition
- Dynamics of Einstein gravity cannot be obtained from an ISO(1,3) action of Yang-Mills type

Conformal gravity in 4d as a gauge theory

- Group parametrizing the symmetry: SO(2,4)
- 15 generators: 6 LT M_{ab}, 4 translations, P_a, 4 conformal boosts K_a and the dilatation D
- Following the same procedure one calculates transf of the gauge fields and tensors after defining the gauge connection
- Action is taken of Yang-Mills form
- Initial symmetry breaks under certain constraints resulting to the *Weyl action*

  Kaku-Townsend-Van Nieu/zen '77,

  Fradkin-Tseytlin '85
- Alternative: Initial symmetry can break down to recover the Einstein action if two specific constraints are taken to hold simultaneously

Chamseddine '02

Construction of a covariant noncommutative space

Kimura '02, Heckman-Verlinde '15

Sperling-Steinacker '17

- dS_4 : homogeneous spacetime with constant curvature (positive)
- Described by the embedding $\eta^{AB} X_A X_B = R^2$ into $\mathrm{M}^{1,4}$
- We aim for a fuzzy dS_4 space
- Coordinates must satisfy $[X_a, X_b] = i\theta_{ab}$, with θ_{ab} remaining to be determined
- Analogy to the fuzzy sphere case suggests identification of the coordinates with generators of the SO(1,4) (isometry group of dS_4)
- BUT: θ_{ab} cannot be assigned to generators of the algebra $\rightarrow$ covariance breaks
- Requiring covariance $\rightarrow$ use a group with larger symmetry $\rightarrow$ minimum extension: SO(1,5) (SO(6) in our language)

- The SO(6) generators, $J_{AB}, A, B = 1, \ldots, 6$, satisfy the commutation relation:

$$[J_{AB}, J_{CD}] = i(\delta_{AC} J_{BD} + \delta_{BD} J_{AC} - \delta_{BC} J_{AD} - \delta_{AD} J_{BC})$$

- After decomposition in SO(4) notation, we identify the component generators as:

$$J_{ab} = \frac{1}{\hbar}\Theta_{ab}, \quad J_{a5} = \frac{1}{\lambda} X_a, \quad J_{a6} = \frac{\lambda}{2\hbar} P_a, \quad J_{56} = \frac{1}{2}\mathrm{h}\,,$$

where X_a, P_a and Θ_{ab} are the coordinate, momentum and noncommutativity tensors, respectively

- The above generators satisfy the following algebra:

$$[X_a, X_b] = i\frac{\lambda^2}{\hbar}\Theta_{ab}, \quad [P_a, P_b] = 4i\frac{\hbar}{\lambda^2}\Theta_{ab}$$

$$[X_a, P_b] = i\hbar\delta_{ab}\mathrm{h}, \quad [X_a, \mathrm{h}] = i\frac{\lambda^2}{\hbar}P_a, \quad [P_a, \mathrm{h}] = 4i\frac{\hbar}{\lambda^2}X_a$$

- Therefore, coordinates are identified as SO(6) generators represented in a (large N) representation

Noncommutative gauge theory of 4d gravity

- Formulation of gravity on the above space
- Noncommutative gauge theory toolbox + the procedure described in the Einstein gravity case

Kimura '02, Heckman-Verlinde '15

- Gauge the isometry group of the space, SO(5) as seen as a subgroup of the SO(6) we ended up
- Anticommutators do not close $\rightarrow$ enlargement of the algebra + fix the representation *Aschieri-Castellani '09*

L. Jonke, A. Chatz/dis, D. Jurman, GM, P. Manousselis, G. Zoupanos '18

- Noncommutative gauge theory of SO(6)xU(1) ($\sim$ U(4)) in the 4 representation
- The generators of the group are represented by combinations of the 4x4 gamma matrices in euclidean signature

- Specifically, the 4x4 generators of the SO(6) group are:
  - six Lorentz rotation generators:
    $\mathrm{M}_{ab} = -\frac{i}{4}[\Gamma_a, \Gamma_b] = -\frac{i}{2}\Gamma_a\Gamma_b \, , a < b$
  - four generators for conformal boosts: $\mathrm{K}_a = \frac{1}{2}\Gamma_a$
  - four generators for translations: $\mathrm{P}_a = -\frac{i}{2}\Gamma_a\Gamma_5$
  - one generator for special conformal transformations:
    $\mathrm{D} = -\frac{1}{2}\Gamma_5$

  Also the following is included
  - one U(1) generator: 1
- From the construction of the above generators from the Γ matrices, we can calculate the algebra they satisfy:

$$[M_{ab}, M_{cd}] = i(\delta_{ac}M_{bd} + \delta_{bd}M_{ac} - \delta_{bc}M_{ad}\delta_{ad}M_{bc}) \, , [K_a, P_b] = i\delta_{ab}D$$
$$[K_a, K_b] = iM_{ab} \, , [P_a, P_b] = iM_{ab} \, , [P_a, D] = iK_a \, , [K_a, D] = -iP_a$$
$$[K_a, M_{bc}] = i(\delta_{ac}K_b - \delta_{ab}K_c) \, , [P_a, M_{bc}] = i(\delta_{ac}P_b - \delta_{ab}P_c) \, , [D, M_{ab}] = 0$$

- Gauging procedure → definition of the covariant coordinate: $\hat{X} = X_m \otimes 1 + A_m(X)$
- gauge connection $\mathrm{A}_m(X)$ taking values in the algebra:

$$A_m(X) = e_m{}^a(X)\otimes P_a + \omega_m{}^{ab}(X)\otimes M_{ab}(X) + b_m{}^a(X)\otimes K_a(X) + \tilde{a}_m(X)\otimes D + a_m(X)\otimes 1$$

- Introduced a gauge field for each generator
- Consider a gauge parameter $\epsilon(x)$:

$$\epsilon = \epsilon_0(X)\otimes 1 + \xi^a(X)\otimes K_a + \tilde{\epsilon}_0(X)\otimes D + \lambda^{ab}(X)\otimes M_{ab} + \tilde{\xi}^a(X)\otimes P_a$$

- Determine the field strength tensor:

$$\mathcal{R}_{mn} = [\hat{X}_m, \hat{X}_n] - \frac{i\lambda^2}{\hbar}\hat{\Theta}_{mn}\otimes 1\,,$$

where $\hat{\Theta}_{mn} = \Theta_{mn}\otimes 1 + \mathcal{B}_{mn}$, where $\mathcal{B}_{mn}$ is a 2-form gauge field valued in U(4) transforming covariantly

- The field strength tensor is also valued in the algebra → it is spanned on the generators

$$\mathcal{R}_{mn}(X) = R_{mn}{}^{ab}\otimes M_{ab} + \tilde{R}_{mn}{}^a\otimes P_a + R_{mn}{}^a\otimes K_a + \tilde{R}_{mn}\otimes D + R_{mn}\otimes 1$$

- The necessary information for calculating the transformations of the fields and the component tensors is in this slide!

- Calculative convenience: employ the SO(5) notation (indices) and then return to the SO(4) ones
- Generators of $SO(6) \times U(1)$ in specific representation given by 4×4 matrices:

$$1, \quad -\frac{i}{4}[\Gamma_a, \Gamma_b], \quad \frac{1}{2}\Gamma_a, \quad -\frac{1}{2}\Gamma_a\Gamma_5, \quad -\frac{1}{2}\Gamma_5\,.$$

- Turning to SO(5) notation: introduce Γ_A with $A, B = 1 \ldots 5$, satisfying:

$$\{\Gamma_A, \Gamma_B\} = 2\delta_{AB}1$$

- Above generators can be written as:

$$1, \quad \Gamma_A, \quad M_{AB} = -\frac{i}{4}[\Gamma_A, \Gamma_B]$$

- Generators in SO(5) notation satisfy the following commutation and anticommutation relations: *Smolin '03*

$$[M_{AB}, M_{CD}] = i(\delta_{AC}M_{BD} + \delta_{BD}M_{AC} - \delta_{BC}M_{AD} - \delta_{AD}M_{BC})$$
$$[\Gamma_M, M_{NP}] = i(\delta_{MP}\Gamma_N - \delta_{MN}\Gamma_P)$$
$$\{M_{AB}, \Gamma_C\} = \epsilon_{ABCDE}M_{DE}$$
$$\{M_{AB}, M_{CD}\} = \frac{1}{2}(\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC})1 + \epsilon_{ABCDE}\Gamma_E$$

- In turn, the covariant coordinate is written as:

$$\hat{X}_m = X_m \otimes 1 + A_m(X) \otimes 1 + A_m{}^{B}(X) \otimes \Gamma_B + A_m{}^{AB}(X) \otimes M_{AB}$$

and the gauge parameter as:

$$\epsilon(X) = \epsilon_0(X) \otimes 1 + A_m(X) \otimes \mathbb{1} + A_m^A(X) \otimes \Gamma_A + A_m^{AB}(X) \otimes M_{AB}$$

- The definition of the field strength tensor is:

$$\hat{F}_{mn} = [\hat{X}_m, \hat{X}_n] - \frac{i\hbar}{\lambda^2}\hat{\Theta}_{mn} \otimes 1$$

- and is also decomposed on the generators:

$$\hat{F}_{mn} = F_{mn}(1) \otimes 1 + F_{mn}{}^{A}(\Gamma_A) \otimes \Gamma_A + F_{mn}{}^{AB}(M_{AB}) \otimes M_{AB}$$

- Transformation rule of the covariant coordinate: $\delta\hat{X}_m = i[\epsilon, \hat{X}_m] \rightarrow$ transformations of the gauge fields (SO(5) notation):

$$\delta A_m \otimes 1 = \left(-i[X_m, \epsilon_0] - i[A_m, \epsilon_0] + i[\xi_A, A_m^A] + \frac{i}{2}[\lambda_{AB}, A_m^{AB}]\right) \otimes 1$$

$$\delta A_m^A \otimes \Gamma_A = \Big(-i[X_m, \xi^A] - i[A_m, \xi^A] + i[\epsilon_0, A_m^A] - \{\xi_B, A_m^{AB}\} + \{\lambda^A{}_B, A_m^B\}$$
$$+ \frac{i}{2}[\lambda^{BC}, A_m^{DE}]\epsilon_{ABCDE}\Big) \otimes \Gamma_A \,,$$

$$\delta A_m^{AB} \otimes M_{AB} = \Big(-i[X_m, \lambda^{AB}] - i[A_m, \lambda^{AB}] + i[\epsilon_0, A_m^{AB}] - 2\{\xi^A, A_m^B\}$$
$$+ \frac{i}{2}[\xi^C, A_m^{DE}]\epsilon_{ABCDE} + \frac{i}{2}[\lambda^{CD}, A_m^E]\epsilon_{ABCDE} - \frac{1}{2}\{\lambda^A_C, A_m^{BC}\}\Big) \otimes M_{AB}$$

- The component tensors (SO(5) notation):

$$F_{mn} \otimes 1 = \Big([X_m, A_n] - [X_n, A_m] + [A_m, A_n] + [A^A_m, A_{nA}] + \frac{1}{2}[A^{AB}_m, A_{nAB}] - \frac{i\hbar}{\lambda^2} B$$

$$F_{mn}{}^A(\Gamma_A) \otimes \Gamma_A = \Big([X_m, A^A_n] + [A_m, A^A_n] - [X_n, A^A_m] - [A_n, A^A_m] + i\{A_{mB}, A^{AB}_n\}$$

$$-i\{A^{AB}_m, A_{nB}\} - \frac{1}{2}\epsilon_{ABCDE}[A^{EB}_m, A^{CD}_n] - \frac{i\hbar}{\lambda^2} B_{mn}{}^A \Big) \otimes \Gamma_A$$

$$F_{mn}{}^{AB}(M_{AB}) \otimes M_{AB} = \Big([X_m, A^{AB}_n] + [A_m, A^{AB}_n] - [X_n, A^{AB}_m] - [A_n, A^{AB}_m] + 2i\{A$$

$$+\frac{1}{2}([A^C_m, A^{DE}_n] - [A^C_n, A^{DE}_m])\epsilon_{ABCDE} + 2i\{A^{AC}_m, A^B_n{}_C\} - \frac{i\hbar}{\lambda^2} B_{mn}{}^{AB} \Big) \otimes M_{AB}$$

- Next, in order to return to the SO(4) notation and give the above results in the SO(4) desirable language, we make the following decompositions of the SO(5) generators:

$$\Gamma_A \to \Gamma_a, \Gamma_5,\ M_{AB} \to M_{ab}, M_{a5},\ B_{mn}{}^{AB} \to B_{mn}{}^{ab}, B_{mn}{}^{a5},\ B_{mn}{}^{A} \to B_{mn}{}^{a}, B_{mn}{}^{5}$$

- Identification of the SO(5) component generators with the SO(4) ones is given by:

$$M_{ab} = -\frac{i}{4}[\Gamma_a, \Gamma_b]\,, \;\; M_{a5} = -\frac{i}{2}\Gamma_a\Gamma_5 = P_a\,, \;\; \Gamma_a = 2K_a\,, \;\; \Gamma_5 = -2D\,, \;\; 1$$

- Accordingly, we decompose the gauge fields to the SO(4) notation:

$$A_m^{AB} \to (A_m^{ab} \equiv \omega_m^{ab}, A_m^{a5} \equiv e_m{}^a)\,, \; A_m^A \to (A_m^a \equiv b_m{}^a, A_m^5 \equiv \tilde{a}_m)\,, \; A_m \to a_m$$

- Also for the components of the SO(5) gauge parameter:

$$\lambda^{AB} \to (\lambda^{ab}, \lambda^{a5} \equiv \tilde{\xi}^a)\,, \quad \xi^A \to (\xi^a, \xi^5 \equiv \tilde{\epsilon}_0)\,, \quad \epsilon_0 \to \epsilon_0$$

- The following results of the transformations of the fields and their curvatures are retrieved -up to some tuning of the numerical coefficients- after considering the commutative limit.

Taking into consideration the above decompositions and identifications we obtain transformations of the fields:

$$\delta\omega_m{}^{ab} = -i[X_m, \lambda^{ab}] - i[a_m, \lambda^{ab}] + i[\epsilon_0, \omega_m{}^{ab}] - 2\{\xi^a, b_m{}^b\} - \frac{1}{2}\{\lambda^a{}_c, \omega_m{}^{bc}\}$$
$$- \frac{1}{2}\{\tilde{\xi}^a, e_m{}^b\} + i[\xi^c, e_m{}^d]\epsilon_{abcd} + \frac{i}{2}[\tilde{\epsilon}_0, \omega_m{}^{cd}]\epsilon_{abcd} + \frac{i}{2}[\lambda^{cd}, \tilde{a}_m]\epsilon_{abcd} - i[\tilde{\xi}^c, b_m{}^d]\epsilon_{abcd}$$
$$\delta e_m{}^a = -i[X_m, \tilde{\xi}^a] - i[a_m, \tilde{\xi}^a] + i[\epsilon_0, e_m{}^a] - \{\xi^a, \tilde{a}_m\} + \{\tilde{\epsilon}_0, b_m{}^a\} + \frac{1}{4}\{\lambda^a{}_b, e_m{}^b\}$$
$$- \frac{1}{4}\{\tilde{\xi}_b, \omega_m{}^{ab}\} + i[\xi^c, \omega_m{}^{bd}]\epsilon_{abcd} - i[\lambda^{cd}, b_m{}^b]\epsilon_{abcd}$$
$$\delta b_m{}^a = -i[X_m, \xi^a] - i[a_m, \xi^a] + i[\epsilon_0, b_m{}^a] - \{\xi_b, \omega_m{}^{ab}\} - 2\{\tilde{\epsilon}_0, e_m{}^a\} + \frac{1}{2}\{\lambda^a{}_b, b_m{}^b\}$$
$$+ \{\tilde{\xi}^a, \tilde{a}_m\} + i[\lambda^{bc}, e_m{}^d]\epsilon_{abcd} + i[\tilde{\xi}^b, \omega_m{}^{cd}]\epsilon_{abcd}$$
$$\delta a_m = -i[X_m, \epsilon_0] - i[a_m, \epsilon_0] + i[\xi^a, b_m{}^a] + i[\tilde{\epsilon}_0, \tilde{a}_m] + \frac{i}{2}[\lambda_{ab}, \omega_m{}^{ab}] + \frac{i}{2}[\tilde{\xi}_a, e_m{}^a]$$
$$\delta\tilde{a}_m = -i[X_m, \tilde{\epsilon}_0] - i[a_m, \tilde{\epsilon}_0] + i[\epsilon_0, \tilde{a}_m] + \{\xi_a, e_m{}^a\} - \{\tilde{\xi}_a, b_m{}^a\} + \frac{i}{2}[\lambda^{ad}, \omega_m{}^{bc}]\epsilon_{abcd}$$

Transformations of the component of $\mathcal{B}_{mn}$ are calculated as well

The component curvatures:

$$R_{mn} = [X_m, a_n] - [X_n, a_m] + [a_m, a_n] + [b_m^{\ a}, b_{na}] + [\tilde{a}_m, \tilde{a}_n] + \frac{1}{2}[\omega_m^{\ ab}, \omega_{nab}] + [e_{ma}, e_n^{\ a}] - \frac{i\hbar}{\lambda^2} B_{mn}$$

$$\tilde{R}_{mn} = [X_m, \tilde{a}_n] + [a_m, \tilde{a}_n] - [X_n, \tilde{a}_m] - [a_n, \tilde{a}_m] - i\{b_{ma}, e_n^{\ a}\} + i\{b_{na}, e_m^{\ a}\} + \frac{1}{2}\epsilon_{abcd}[\omega_m^{\ ab}, \omega_n^{\ cd}] - \frac{i\hbar}{\lambda^2}\tilde{B}_{mn}$$

$$R_{mn}^{\ \ a} = [X_m, b_n^{\ a}] + [a_m, b_n^{\ a}] - [X_n, b_m^{\ a}] - [a_n, b_m^{\ a}] + i\{b_{mb}, \omega_m^{\ ab}\} - i\{b_{nb}, \omega_m^{\ ab}\} + i\{\tilde{a}_m, e_n^{\ a}\} - i\{\tilde{a}_n, e_m^{\ a}\} + \epsilon_{abcd}([e_m^{\ b}, \omega_n^{\ cd}] - [e_n^{\ b}, \omega_m^{\ cd}]) - \frac{i\hbar}{\lambda^2} B_{mn}^{\ \ a}$$

$$\tilde{R}_{mn}^{\ \ a} = [X_m, e_n^{\ a}] + [a_m, e_n^{\ a}] - [X_n, e_m^{\ a}] - [a_n, e_m^{\ a}] + i\{b_m^{\ a}, \tilde{a}_n\} - i\{b_n^{\ a}, \tilde{a}_m\} - ([b_m^{\ b}, \omega_n^{\ cd}] - [b_n^{\ b}, \omega_m^{\ cd}])\epsilon_{abcd} - i\{\omega_m^{\ ab}, e_{nb}\} + i\{\omega_n^{\ ab}, e_{mb}\} - \frac{i\hbar}{\lambda^2}\tilde{B}_{mn}^{\ \ a}$$

$$R_{mn}^{\ \ ab} = [X_m, \omega_n^{\ ab}] + [a_m, \omega_n^{\ ab}] - [X_n, \omega_m^{\ ab}] - [a_n, \omega_m^{\ ab}] + 2i\{b_m^{\ a}, b_n^{\ b}\} + ([b_m^{\ c}, e_n^{\ d}] - [b_n^{\ c}, e_m^{\ d}])\epsilon_{abcd} + \frac{1}{2}([\tilde{a}_m, \omega_n^{\ cd}] - [\tilde{a}_n, \omega_m^{\ cd}])\epsilon_{abcd} + 2i\{\omega_m^{\ ac}, \omega_n^{\ b}{}_c\} + 2i\{e_m^{\ a}, e_n^{\ b}\} - \frac{i\hbar}{\lambda^2} B_{mn}^{\ \ ab}$$

The constraints for the symmetry breaking

- We want to result with SO(4) symmetry out of SO(6) part
- Employ the constraint of the torsionless condition: $\tilde{R}(P) = 0$
- We adopt: $e_m{}^a = b_m{}^a$ and fix $\tilde{a}_m = 0$
- Solving the constraint we obtain an expression $\omega_m{}^{ab} = \omega_m{}^{ab}(e, a)$:

$$\omega_n{}^{ac} = \frac{3}{4} e^m{}_b \, \epsilon^{abcd} [D_m, e_{nd}]$$

The action of the theory

- The choice of the action is of *Yang-Mills* type:

$$\mathcal{S} = \mathrm{Tr}\mathrm{tr}\mathcal{R}_{mn}\mathcal{R}_{rs}\epsilon^{mnrs}$$

- In the action we have to include the kinetic term of the $\mathcal{B}$ field:

$$\mathcal{S}_{\mathcal{B}} = \mathrm{Tr}\,\mathrm{tr}\,\hat{\mathcal{H}}_{mnp}\hat{\mathcal{H}}^{mnp}$$

- where $\hat{\mathcal{H}}$ is the field strength tensor of the $\mathcal{B}$ field:

$$\hat{\mathcal{H}}_{mnp} = \frac{1}{3}\left([\hat{X}_m, \hat{\Theta}_{np}] + [\hat{X}_n, \hat{\Theta}_{pm}] + [\hat{X}_p, \hat{\Theta}_{mn}]\right)$$

- Using the reduced expressions of the tensors, that is taking the constraints into consideration, we will obtain the equations of motion after variation of the above action

Summary

- 4d Einstein gravity described as a gauge theory
- Formulate a noncommutative covariant space
- Gauge theory of the (extended) isometry group
- Obtain the transformations of the fields
- Obtain the component curvatures
- Consider constraints for obtaining the desired symmetry
- Propose an action, obtain the e.o.m. from variation

Thank you for your attention!

- The algebra of the conformal generators:

$$[M_{ab}, M^{cd}] = 4M_{[a}{}^{[d}\delta_{b]}^{c]}\,, \quad [M_{ab}, P_c] = 2P_{[a}\delta_{b]c}\,, \quad [M_{ab}, K_c] = 2K_{[a}\delta_{b]c}\,,$$
$$[P_a, D] = P_a\,, \quad [K_a, D] = -K_a\,, \quad [P_a, K_b] = 2(\delta_{ab}D - M_{ab})\,,$$

where $a, b, c, d = 1 \ldots 4$.

- Definition of the gauge connection:

$$A_\mu = e_\mu{}^a P_a + \frac{1}{2}\omega_\mu{}^{ab}M_{ab} + b_\mu D + f_\mu{}^a K_a$$

in which for every generator, a gauge field has been corresponded.

- The above connection obeys the following infinitesimal transformation rule:

$$\delta_\epsilon A_\mu = D_\mu \epsilon = \partial_\mu \epsilon + [A_\mu, \epsilon]$$

where ϵ is a parameter taking values in the Lie algebra of the conformal group:

$$\epsilon = \epsilon_P{}^a P_a + \frac{1}{2}\epsilon_M{}^{ab}M_{ab} + \epsilon_D D + \epsilon_K{}^a K_a$$

- Transformations of the gauge fields:

$$\begin{aligned}
\delta e_\mu{}^a &= \partial_\mu \epsilon_P^a + 2ie_{\mu b}\epsilon_M^{ab} - i\omega_\mu{}^{ab}\epsilon_{Pb} - b_\mu \epsilon_K^a + f_\mu{}^a \epsilon_D \,, \\
\delta\omega_\mu{}^{ab} &= \frac{1}{2}\partial_\mu \epsilon_M^{ab} + 4ie_\mu{}^a \epsilon_P^b + \frac{i}{4}\omega_\mu{}^{ac}\epsilon_M{}^b{}_c + if_\mu{}^a \epsilon_K^b \,, \\
\delta b_\mu &= \partial_\mu \epsilon_D - e_\mu{}^a \epsilon_{Ka} + f_\mu{}^a \epsilon_{Pa} \,, \\
\delta f_\mu{}^a &= \partial_\mu \epsilon_K^a + 4ie_\mu{}^a \epsilon_D - i\omega_\mu{}^{ab}\epsilon_{Kb} - 4ib_\mu \epsilon_P^a + if_\mu{}^b \epsilon_M{}^a{}_b
\end{aligned}$$

- The component curvature tensors:

$$\begin{aligned}
R_{\mu\nu}{}^a(P) &= 2\partial_{[\mu} e_{\nu]}{}^a + f_{[\mu}{}^a b_{\nu]} + e_{[\mu}{}^b \omega_{\nu]}{}^{ac}\delta_{bc}, \\
R_{\mu\nu}{}^{ab}(M) &= \partial_{[\mu}\omega_{\nu]}{}^{ab} + \omega_{[\mu}{}^{ca}\omega_{\nu]}{}^{db}\delta_{cd} + e_{[\mu}{}^a e_{\nu]}{}^b + f_{[\mu}{}^a f_{\nu]}{}^b, \\
R_{\mu\nu}(D) &= 2\partial_{[\mu} b_{\nu]} + f_{[\mu}{}^a e_{\nu]}{}^b \delta_{ab}, \\
R_{\mu\nu}{}^a(K) &= 2\partial_{[\mu} f_{\nu]}{}^a + e_{[\mu}{}^a b_{\nu]} + f_{[\mu}{}^b \omega_{\nu]}{}^{ac}\delta_{bc} \,.
\end{aligned}$$