Localizing gravity on a Hyperbolic Braneworld

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A. Salam & E. Sezgin, Phys.Lett. B147 (1984) 47
M. Cvetič, G. Gibbons & C.N. Pope, Nucl. Phys. B677 (2004) 164
T. Pugh, E. Sezgin & K.S.S., JHEP 1102 (2011) 115
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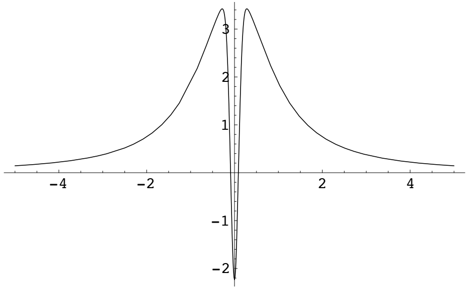
Braneworld localized gravity

The idea of formulating the cosmology of our universe on a brane embedded in a higher-dimensional spacetime dates back, at least, to Rubakov and Shaposhnikov. Phys. Lett. B125 (1983), 136

Localizing gravity with an infinite transverse space

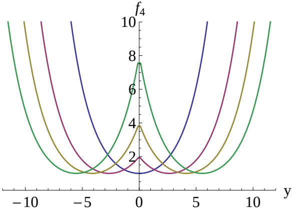
Attempts in a supergravity context to achieve a localization of gravity on a brane embedded in an infinite transverse space were made by Randall and Sundrum (RS II) Phys. Rev. Lett. 83 (1999) 4690 and by Karch and Randall JHEP 0105 (2001) 008 using patched-together sections of AdS_5 space with a delta-function source at the joining surface. This produced a "volcano potential" for the effective Schrödinger problem in the direction transverse to the brane, giving rise to a bound state concentrating gravity in the 4D directions.



General problems with localization

Attempting to embed such models into a full supergravity/string-theory context have proved to be problematic, however. Splicing together sections of AdS_5 is clearly an artificial construction that does not make use of the natural D-brane or NS-brane objects of string or supergravity theory.

These difficulties were studied more generally by Bachas and Estes JHEP 1106 (2011) 005 , who traced the difficulty in obtaining localization within a string or supergravity context to the behavior of the warp factor for the 4D subspace. In the Karch-Randall spliced model, one obtains a peak in the warp factor at the junction:



Bachas & Estes no-go argument

Here's why Bachas and Estes considered that one could not have a natural localization of gravity on a brane with an infinite transverse space. Consider fluctuations away from a smooth D-dim background $d\hat{s}^2 = e^{2A(z)}(\eta_{\mu\nu} + h_{\mu\nu}(x)\xi(z))dx^\mu dx^\nu + \hat{g}_{ab}(z)dz^a dz^b$ where $\xi(z)$ is the transverse wave function. Such a transverse wave function with eigenvalue λ needs to satisfy the transverse wave equation $\frac{e^{-2A}}{\sqrt{\hat{g}}}\partial_a(\sqrt{\hat{g}}e^{4A}\hat{g}^{ab}\partial_b)\xi = -\lambda\xi$. The norm of $\xi(z)$ is then given by $\lambda||\xi||^2 = -\int d^{D-4}z\xi(\partial_a\sqrt{\hat{g}}e^{4A}\hat{g}^{ab}\partial_b\xi)$.

If one *assumes* that one may integrate by parts without producing a surface term, one would have $\lambda||\xi||^2 \longrightarrow \int d^{D-4}z\sqrt{\hat{g}}e^{4A}|\partial\xi|^2$. Then, if one is looking for a transverse wavefunction ξ with $\lambda = 0$ as needed for massless 4D gravity, one would need $\partial_a\xi = 0$ yielding $\xi = \mathrm{constant}$, which is not normalizable in an infinite transverse space.

The resolution of this problem requires very specific self-adjointness features of the transverse wavefunction problem.

Another approach: Salam-Sezgin theory and its embedding

Abdus Salam and Ergin Sezgin constructed in 1984 a version of 6D minimal (chiral, *i.e.* (1,0)) supergravity coupled to a 6D 2-form tensor multiplet and a 6D super-Maxwell multiplet which gauges the U(1) R-symmetry of the theory. Phys.Lett. B147 (1984) 47 This Einstein-tensor-Maxwell system has the bosonic Lagrangian

$$\begin{aligned} \mathcal{L}_{\mathrm{SS}} &= \tfrac{1}{2}R - \tfrac{1}{4g^2}e^{\sigma}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{6}e^{-2\sigma}G_{\mu\nu\rho}G^{\mu\nu\rho} - \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - g^2 e^{-\sigma} \\ G_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + 3F_{[\mu\nu}A_{\rho]} \end{aligned}$$

Note the *positive* potential term for the scalar field σ. This is a key feature of all R-symmetry gauged models generalizing the Salam-Sezgin model, leading to models with noncompact symmetries. For example, upon coupling to yet more vector multiplets, the sigma-model target space can have a structure $SO(p,q)/(SO(p)\times SO(q))$.

The Salam-Sezgin theory does not admit a maximally symmetric 6D solution, but it does admit a $(\text{Minkowski})_4 \times S^2$ solution with the flux for a $U(1)$ monopole turned on in the S^2 directions

$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + a^2(d\theta^2 + \sin^2\theta d\phi^2) \\
A_m dy^m &= (n/2g)(\cos\theta \mp 1)d\phi \\
\sigma &= \sigma_0 = \text{const} \,, \qquad B_{\mu\nu} = 0 \\
g^2 &= \frac{e^{\sigma_0}}{2a^2} \,, \qquad n = \pm 1
\end{aligned}
$$

This construction has been used in the SLED $\leftrightarrow$ Supersymmetry in Large Extra Dimensions proposal for dilution of the cosmological constant in the two extra S^2 dimensions, leaving a naturally small residue in the four x^μ dimensions.

Aghababaie, Burgess, Parameswaran & Quevedo, Nucl. Phys. B680 (2004) 389

$\mathcal{H}^{(2,2)}$ embedding of the Salam-Sezgin theory

A way to obtain the Salam-Sezgin theory from M theory was given by Cvetič, Gibbons & Pope. Nucl. Phys. B677 (2004) 164 This employed a reduction from 10D type IIA supergravity on the space $\mathcal{H}^{(2,2)}$, or, equivalently, from 11D supergravity on $S^1 \times \mathcal{H}^{(2,2)}$. The $\mathcal{H}^{(2,2)}$ space is a cohomogeneity-one 3D hyperbolic space which can be obtained by embedding into R^4 via the condition $\mu_1^2 + \mu_2^2 - \mu_3^2 - \mu_4^2 = 1$. This embedding condition is $\mathrm{SO}(2,2)$ invariant, but the embedding $\mathbb{R}^4$ space has SO(4) symmetry, so the isometries of this space are just $\mathrm{SO}(2,2) \cap \mathrm{SO}(4) = \mathrm{SO}(2) \times \mathrm{SO}(2)$. The cohomogeneity-one $\mathcal{H}^{(2,2)}$ metric is $ds_3^2 = \cosh 2\rho d\rho^2 + \cosh^2\rho d\alpha^2 + \sinh^2\rho d\beta^2$.

Since $\mathcal{H}^{(2,2)}$ admits a natural $SO(2,2)$ group action, the resulting 7D supergravity theory has maximal (32 supercharge) supersymmetry and a gauged $SO(2,2)$ symmetry, linearly realized on $SO(2) \times SO(2)$. Note how this fits neatly into the general scheme of extended Salam-Sezgin gauged models.

The $\mathcal{H}^{(2,2)}$ reduced theory in 7D can be further truncated to minimal (16 supercharge) 7D supersymmetry, then yet further reduced on $S^1/\mathbb{Z}_2$ to obtain precisely the $(1,0)$ 6D Salam-Sezgin gauged $U(1)$ supergravity theory. This naturally admits the $(\text{Minkowski})_4 \times S^2$ Salam-Sezgin "ground state" solution. Moreover, the result of this chain of reductions from 11D or 10D is a mathematically consistent truncation: every solution of the 6D Salam-Sezgin theory can be lifted to an exact solution in 10D type IIA or 11D supergravity.

The Kaluza-Klein spectrum

Reduction on the non-compact $\mathcal{H}^{(2,2)}$ space from ten to seven dimensions, despite its mathematical consistency, does not provide a full physical basis for compactification to 4D. The chief problem is that the truncated Kaluza-Klein modes form a *continuum* instead of a discrete set with mass gaps. Moreover, the wavefunction of "reduced" 4D states when viewed from 10D or 11D includes a non-normalizable factor owing to the infinite $\mathcal{H}^{(2,2)}$ directions. Accordingly, the higher-dimensional supergravity theory does not naturally localize gravity in the lower-dimensional subspace when handled by ordinary Kaluza-Klein methods.

Expansion about the Salam-Sezgin background

The $D = 10$ lift of the Salam-Sezgin "vacuum" solution yields the metric

$$\begin{aligned} ds_{10}^2 &= (\cosh 2\rho)^{1/4} \Big[e^{-\frac{1}{4}\bar\phi}\, d\bar s_6^2 + e^{\frac{1}{4}\bar\phi}\, dy^2 + \tfrac{1}{2}\bar g^{-2}\, e^{\frac{1}{4}\bar\phi} \Big(d\rho^2 \\ &\quad + \tfrac{1}{4}[d\psi + \mathrm{sech} 2\rho (d\chi - 2\bar g \bar A)]^2 + \tfrac{1}{4}(\tanh 2\rho)^2\, (d\chi - 2\bar g \bar A)^2 \Big) \Big] \\ \bar A_{(1)} &= -\frac{1}{2\bar g}\, \cos\theta\, d\varphi \end{aligned}$$

in which the $d\bar s_6^2$ metric has $\text{Minkowski}_4 \times S^2$ structure

$$d\bar s_6^2 = dx^\mu dx^\nu \eta_{\mu\nu} + \frac{1}{8\bar g^2}\,(d\theta^2 + \sin^2\theta d\varphi^2)$$

The inclusion of gravitational fluctuations about this background is then accomplished by replacing

$$\eta_{\mu\nu} \longrightarrow \eta_{\mu\nu} + h_{\mu\nu}(x, z)$$

where z^p are the coordinates transverse to the 4D coordinates x^μ.

Bound states and mass gaps Crampton, Pope & K.S.S.

An approach to obtaining the localization of gravity on the 4D subspace is then to look for a *normalizable* transverse-space wavefunction $\xi(z)$ for $h_{\mu\nu}(x,z) = h_{\mu\nu}(x)\xi(z)$ with a mass gap before the onset of the continuous massive Kaluza-Klein spectrum. This could be viewed as analogous to an effective field theory for electrons confined to a metal by a nonzero work function.

General study of the fluctuation spectra about brane solutions shows that the mass spectrum of the spin-two fluctuations about a brane background is given by the spectrum of the scalar Laplacian in the transverse embedding space of the brane

Csaki, Erlich, Hollowood & Shirman, Nucl.Phys. B581 (2000) 309; Bachas & Estes, JHEP 1106 (2011) 005

$$
\begin{aligned}
\Box_{(10)} F &= \frac{1}{\sqrt{-\det g_{(10)}}}\partial_M\left(\sqrt{-\det g_{(10)}}\,g_{(10)}^{MN}\partial_N F\right)\\
&= H_{SS}^{\frac{1}{4}}(\Box_{(4)} + g^2\triangle_{\theta,\phi,y,\psi,\chi} + g^2\triangle_{KK})\\
H_{SS} &= (\cosh 2\rho)^{-1}\ \text{warp factor;}\quad \triangle_{KK} = \frac{\partial^2}{\partial\rho^2} + \frac{2}{\tanh(2\rho)}\frac{\partial}{\partial\rho}
\end{aligned}
$$

The z^p directions θ, ϕ, y, ψ & χ are all compact, and one can employ ordinary Kaluza-Klein methods for reduction on them, truncating to the invariant sector for these coordinates, but still allowing dependence on the noncompact coordinate ρ.

To handle the noncompact direction ρ, one needs to expand all fields in eigenmodes of $\triangle_{\mathsf{KK}}$:

$$\phi(x^\mu, \rho) = \sum_i \phi_{\lambda_i}(x^\mu)\xi_{\lambda_i}(\rho) + \int_\Lambda^\infty d\lambda \phi_\lambda(x^\mu)\xi_\lambda(\rho)$$

where the ϕ_{λ_i} are discrete eigenmodes and the ϕ_λ are the continuous Kaluza-Klein eigenmodes. Their eigenvalues give the Kaluza-Klein masses in 4D from $\Box_{(10)}\phi_\lambda = 0$ using $\triangle_{\theta,\phi,y,\psi,\chi}\phi_\lambda = 0$:

$$\begin{aligned} \triangle_{\mathsf{KK}}\xi_\lambda &= -\lambda\xi_\lambda \\ \Box_{(4)}\phi_\lambda &= (g^2\lambda)\phi_\lambda \end{aligned}$$

The Schrödinger equation for $\mathcal{H}^{(2,2)}$ eigenfunctions

One can rewrite the $\triangle_{\mathrm{KK}}$ eigenvalue problem as a Schrödinger equation by making the substitution

$$\Psi_\lambda = \sqrt{\sinh(2\rho)}\xi_\lambda$$

after which the eigenfunction equation takes the Schrödinger equation form

$$-\frac{d^2\Psi_\lambda}{d\rho^2} + V(\rho)\Psi_\lambda = \lambda\Psi_\lambda$$

where the potential is

$$V(\rho) = 2 - \frac{1}{\tanh^2(2\rho)}$$

The SS Schrödinger equation potential $V(\rho)$ asymptotes to the value 1 for large ρ. In this limit, the Schrödinger equation becomes

$$\frac{d^2\Psi_\lambda}{d\rho^2} + 4e^{-4\rho}\Psi_\lambda + (\lambda - 1)\Psi_\lambda = 0$$

giving scattering-state solutions for $\lambda > 1$:

$$\Psi_\lambda(\rho) \sim \left(A_\lambda e^{i\sqrt{\lambda-1}\rho} + B_\lambda e^{-i\sqrt{\lambda-1}\rho}\right) \qquad \text{for large } \rho$$

while for $\lambda < 1$, one can have L^2 normalizable bound states. Recalling the ρ dependence of the measure $\sqrt{-g_{(10)}} \sim (\cosh(2\rho))^{\frac{1}{4}} \sinh(2\rho)$, one finds for large ρ

$$\int_{\rho_1 \gg 1}^{\infty} |\Psi_\lambda(\rho)|^2 d\rho < \infty \Rightarrow \Psi_\lambda \sim B_\lambda e^{-\sqrt{1-\lambda}\rho} \text{ for } \lambda < 1$$

So for $\lambda < 1$ we have candidates for bound states.

The Schrödinger equation potential

The limit as $\rho \to 0$ of the potential $V(\rho) = 2 - 1/\tanh^2(2\rho)$ is just $V(\rho) = -1/(4\rho^2)$. The associated Schrödinger problem has a long history as one of the most puzzling cases in one-dimensional quantum mechanics. It has been studied and commented upon by Von Neumann; Pauli; Case; Landau & Lifshitz; de Alfaro, Fubini & Furlan, and many others.

The $-1/4$ coefficient is key to the peculiarity of this Schrödinger problem: for coefficients $\alpha > -1/4$, there is no L^2 normalizable ground state, while for $\alpha < -1/4$, an infinity of L^2 normalizable discrete bound states appear.

For the precise coefficient $\alpha = -1/4$, a regularized treatment shows the existence of a *single* L^2 normalizable bound state separated by a mass gap and lying below the continuum of scattering states. A.M. Essin & D.J. Griffiths, Am.J. Phys. 74, 109 (2006) The precise eigenvalue of this ground state, however, is not fixed by normalizability considerations and hence remains, so far, a free parameter of the quantum theory.

The zero-mode bound state

The Schrödinger potential $V(\rho) = 2 - \coth^2(2\rho)$ diverges as $\rho \to 0$; this is a regular singular point of the Schrödinger equation. Near $\rho = 0$, solutions have a structure given by a Frobenius expansion

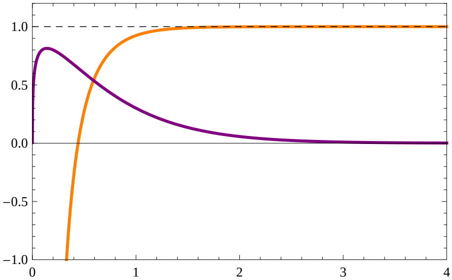
$$\Psi_\lambda \sim \sqrt{\rho}(C_\lambda + D_\lambda \log \rho)$$

This behavior at the origin does not affect L^2 normalizability, but it does indicate that we have a family of candidate bound states characterized by $\theta = \arctan(\frac{C_\lambda}{D_\lambda})$.

The 1-D quantum mechanical system with a $V(\rho) = 2 - \coth^2(2\rho)$ potential belongs to a special class of Pöschl-Teller integrable systems. Neither normalizability nor self-adjointness are by themselves sufficient to completely determine the transverse wavefunction for the reduced effective theory, *i.e.* the value of the parameter θ. A key feature of such systems, however is 1-D *supersymmetry* and requiring that this be unbroken by the transverse wavefunction Ψ_λ selects the value $\lambda = 0$.

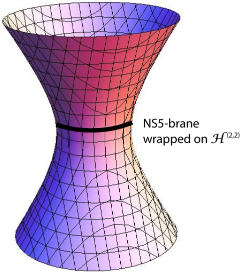
Happily, for $\lambda = 0$ the Schrödinger equation can be solved exactly. The normalized result, corresponding to $\theta = 0$, is

$$\Psi_0(\rho) = \sqrt{\sinh(2\rho)}\xi_0(\rho) = \frac{2\sqrt{3}}{\pi}\sqrt{\sinh(2\rho)}\log(\tanh\rho)$$



$\mathcal{H}^{(2,2)}$ Schrödinger equation potential (orange) and zero-mode ξ_0 (purple)

The asymptotic structure of the Salam-Sezgin background as $\rho \to 0$ limits to the horizon structure of a NS-5 brane. This also allows for the inclusion of an additional NS-5 brane source as $\rho \to 0$. After such an inclusion, the zero-mode transverse wavefunction ξ_0 remains *unchanged*. Moreover, inclusion of such an additional NS-5 brane does not alter the 8 unbroken space-time supersymmetries possessed by the Salam-Sezgin background. The NS-5 modified 10-D supergravity solution can still be given explicitly.



$\mathcal{H}^{(2,2)}$ space with an NS-5 brane source wrapped around its 'waist' and smeared on a transverse S^2

Braneworld effective gravity

The effective action for 4D gravity reduced on the background SS solution is obtained by letting the higher dimensional metric take the form $d\hat{s}^2 = e^{2A(z)}(\eta_{\mu\nu} + h_{\mu\nu}(x)\xi_0(\rho))dx^\mu dx^\nu + \hat{g}_{ab}(z)dz^a dz^b$, where the warp factor $A(z)$ and the transverse metric $\hat{g}_{ab}(z)$ are given by the SS background.

Starting from the 10D Einstein gravitational action

$$I_{10} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{\hat{g}}\, \hat{R}(\hat{g})$$

and making the reduction to 4D, one obtains at quadratic order in $h_{\mu\nu}$ the linearized 4D Einstein action with a prefactor υ_0^{-2}

$$I_{\mathrm{lin}\,4} = \frac{1}{\upsilon_0^2} \int d^4x \left(-\tfrac{1}{2}\partial_\sigma h_{\mu\nu}\partial^\sigma h^{\mu\nu} + \tfrac{1}{2}\partial_\mu h^\sigma{}_\sigma \partial^\mu h^\tau{}_\tau + \partial^\nu h_{\mu\nu}\partial^\sigma h^\mu{}_\sigma + h^\sigma{}_\sigma \partial^\mu \partial^\nu h_{\mu\nu}\right)$$

The normalizing factor $\upsilon_0 = \left(\frac{16\pi G_{10} g^5}{\pi^2 \ell_y I_2}\right)^{\frac{1}{2}}$ involves the first of a series of integrals involving products of the transverse wavefunction ξ_0. For υ_0 one needs

$$I_2 = \int_0^\infty d\rho \sinh 2\rho\, \xi_0^2 = \frac{\pi^2}{12}$$

The ability to explicitly evaluate such integrals of products of transverse wave functions is directly related to the integrable-model Pöschl-Teller structure of the transverse wavefunction Schrödinger equation with $V(\rho) = 2 - \coth^2(2\rho)$. This is reminiscent of the way in which analogous integrals for the hydrogen atom can be evaluated using the integrable structure following from its SO(4) symmetry. M. Lieber, Phys.Rev. 174 (1968) 203

In order to obtain the effective 4D Newton's constant, one needs to rescale $h_{\mu\nu} = \upsilon_0 \tilde{h}_{\mu\nu}$ in order to obtain a canonically-normalised kinetic term for $\tilde{h}_{\mu\nu}$. Then the leading effective 4D coupling $\kappa_4 = \sqrt{32\pi G_4}$ for gravitational self-interactions is obtained from the coefficient in front of the trilinear terms in $\tilde{h}_{\mu\nu}$ in the 4D effective action.

These involve the integral

$$I_3 = \int_0^\infty d\rho \sinh 2\rho\, \xi_0^3 = -\frac{3\zeta(3)}{4} \ ;$$

accordingly, the 4D Newton constant is given by

$$G_4 = \frac{486\, \zeta(3)^2 G_{10} g^5}{\pi^8 \ell_y}$$

with corresponding 4D expansion coupling

$$\kappa_4 = 72\sqrt{3}\zeta(3) \left(\frac{G_{10} g^5}{\pi^7 \ell_y} \right)^{\frac{1}{2}} .$$

Inconsistent truncations and effective theory corrections

Note that the convergence of the I_2 and I_3 integrals in the evaluation of G_4 is ensured by the presence of the $\sinh 2\rho$ factor as $\rho \to 0$ and by the asymptotic falloff of $\xi_0(\rho)$ as $\rho \to \infty$.

By contrast, in a standard Kaluza-Klein toroidal reduction to 4D, the transverse wavefunction would just be $\xi = \mathrm{const}$, causing the I_2 integral to diverge. This would, however, give rise to a *vanishing* 4D Newton constant. The standard Kaluza-Klein reduction yields a consistent truncation Cvetič, Gibbons & Pope (2004) , but the price one pays for this with an infinte transverse space is to have $G_4^{\xi\,\mathrm{const}} = 0$.

The reduction with a normalizable transverse wavefunction $\xi_0(\rho)$ yields an acceptably finite G_4, but at the price that the reduction does not produce a consistent truncation. This can be thought of as a feature rather than a bug, however, as what it means is that instead of suppressing the massive Kaluza-Klein modes, one should properly integrate them out in evaluating the 4D effective theory.

The Pöschl-Teller integrable structure of the transverse Schrödinger problem enables much of this to be done explicitly. The other ξ_0^n integrals needed in evaluating the leading effective theory can also be done explicitly. One finds

$$I_n \equiv \int_0^\infty d\rho \, \sinh 2\rho \, \xi_0^n(\rho) = (-1)^n \, n! \, 2^{-n} \, \zeta(n)$$

Moreover, integrating out the continuum of massive modes also requires performing integrals like

$$\int_0^\infty d\rho \, \sinh 2\rho \, \xi_0^n(\rho) \xi_\lambda(\rho)$$

which can also be evaluated and the results given in terms of Legendre functions. Integrating out the ξ_λ contributions then produces a series of corrections to the leading-order effective theory.

Life in an inconsistent truncation

The "inconsistency" of the reduction to $D = 4$ is revealed in the types of corrections to the lower-dimensional effective theory that can arise from integrating out the massive modes.

There are some similarities here to compactification on Calabi-Yau spaces. M.J. Duff, S. Ferrara, C.N. Pope & K.S.S., Nucl.Phys. B333 (1990) 783 However, in such CY compactifications, if one focuses on parts of the leading order effective theory without scalar potentials, the result of integrating out the massive KK modes is purely to generate higher-derivative corrections to the leading order effective theory.

In the present case, however, important corrections can be obtained also in the leading order two-derivative part of the effective theory. One can see this thanks to the special integrability features of the Pöschl-Teller transverse wavefunctions, which allow for transverse integrals actually to be done explicitly.

Note, for example that quartic terms in $h_{\mu\nu}(x)$ involve the integral $I_4 = 4!\,2^{-4}\,\zeta(4)$. This, however, does not yet yield the expected quartic term with a coefficient $(\kappa_4)^2$: I_4 involves $\zeta(4)$, while $(\kappa_4)^2$ involves $(\zeta(3))^2$.

The deficit has to arise from the result of integrating out massive modes such as the Kaluza-Klein vectors $H_{\mu n_\rho} = V_\mu$. In a standard compactification in which the Kaluza-Klein vectors are all paired up with isometries of the compactifying space, such KK vectors would form part of the massless spectrum, but this is not the case where there is no isometry corresponding to the V_μ.

Conclusions

- Braneworld gravity on a subsurface of the Salam-Sezgin hyperbolic vacuum spacetime can successfully be localized within an infinite transverse space. This is in contrast to the situation with asymptotically maximally symmetric spacetimes where localization has failed.
- There is a mass gap between the zero mode and the edge of the continuous massive spectrum: gravity is localized on the 6D brane worldvolume. Further standard Kaluza-Klein compactification to 4D then gives localized 4D braneworld gravity.
- Such reductions involve inconsistent Kaluza-Klein truncations, but the details of the lower dimensional effective action can nonetheless be worked out thanks to the integrability properties of the equivalent Schrödinger problem for the transverse wave functions. This could lead to interesting braneworld phenomenology.