UNIVERSITY OF ICELAND

Stockholm University

Charged black holes and near AdS_2 holography

Lárus Thorlacius

based on

arXiv:1810.08741
by A. Brown, H. Gharibyan, H. Lin, L. Susskind, LT, Y. Zhao

and (if time permits) on

Phys. Rev. D 98 (2018) 126016
by A. Brown, H. Gharibyan, A. Streicher, L. Susskind, LT, Y. Zhao

Quantum Spacetime '19, Bratislava, 11-15 February 2019

Motivation

1) Explore holographic complexity conjectures ("C = V" and "C = A") in a simple setting

Brown et al., 1810.08741 "The Case of the Missing Gates: Complexity of Jackiw-Teitelboim Gravity"
(see also Goto et al., 1901.00014 "Holographic Complexity Equals Which Action?")

— 1+1-dilaton gravity model

Teitelboim (1993); Jackiw (1995)
Almheiri, Polchinski (2014); Jensen (2016); Engelsöy, Mertens, Verlinde (2016)
Maldacena, Stanford, Yang (2016); Harlow, Jafferis (2018); …..

— broken conformal symmetry —> low-energy dynamics governed by Schwarzian effective action

— the same (broken) symmetry is realized in the SYK model
—> Schwarzian action captures important aspects of SYK dynamics

— SYK model has discrete field variables with q-local Hamiltonian
—> quantum complexity better defined than in continuum QFT

2) Explore conjectured correspondence between operator size in chaotic QFT and radial momentum in bulk dual

Susskind, 1802.01198 "Why do things fall?"
Brown et al., Phys. Rev. D 98 (2018) 126016 "Falling Toward Charged Black Holes"

Holographic quantum complexity

Model BH scrambling dynamics by a quantum circuit with a total number of qubits of order S and a universal set of primitive gates. Hayden, Preskill (2006)

The quantum complexity of a circuit state is the minimum number of primitive gates needed to obtain that state from a given reference state.

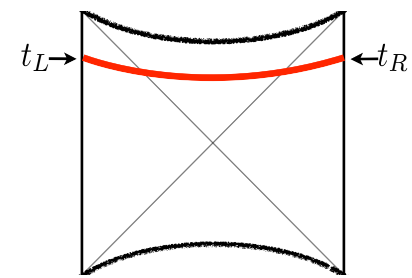
Assuming each qubit gets acted on by at most one primitive gate per cycle we expect $\frac{\Delta \mathcal{C}}{\Delta \tau} \sim S$

or, if each cycle takes of order one unit of Rindler time: $\boxed{\frac{d\mathcal{C}}{dt_S} \sim S\,T}$ $\tau_R = \frac{2\pi}{\beta}\, t_S$

Holographic complexity conjectures:

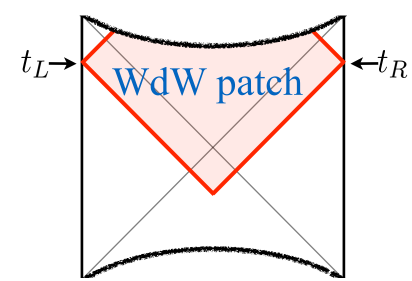
1) Complexity equals volume $\mathcal{C} \sim \frac{V}{G_N R_0}$

Susskind (2014)



2) Complexity equals action $\mathcal{C} = \frac{\mathcal{A}}{\pi}$

Brown, Roberts, Susskind, Swingle, Zhao (2015)



Jackiw-Teitelboim model

Action

$$S_{JT} = \frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{-g}\,\varphi\left(R + \frac{2}{L^2}\right) + \int_{\partial\mathcal{M}} d\tau\,\varphi\left(K + \frac{1}{L}\right)$$

topological term →

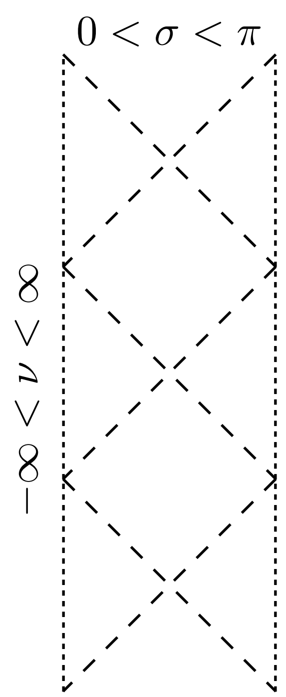
$$+\,\varphi_0\left(\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{-g}\,R + \int_{\partial\mathcal{M}} d\tau\,K\right)$$

Field equations

AdS_2 geometry →

$$0 = R + \frac{2}{L^2}$$

$$0 = \nabla_\alpha \nabla_\beta\,\varphi - g_{\alpha\beta}\left(\nabla^2\varphi - \frac{1}{L^2}\varphi\right)$$



Global coordinates on AdS_2

$$ds^2 = \frac{L^2}{\sin^2\sigma}\left(-\,d\nu^2 + d\sigma^2\right)$$

Dilaton field

$$\varphi(\nu,\sigma) = \varphi_H\,\frac{\cos\nu}{\sin\sigma}$$

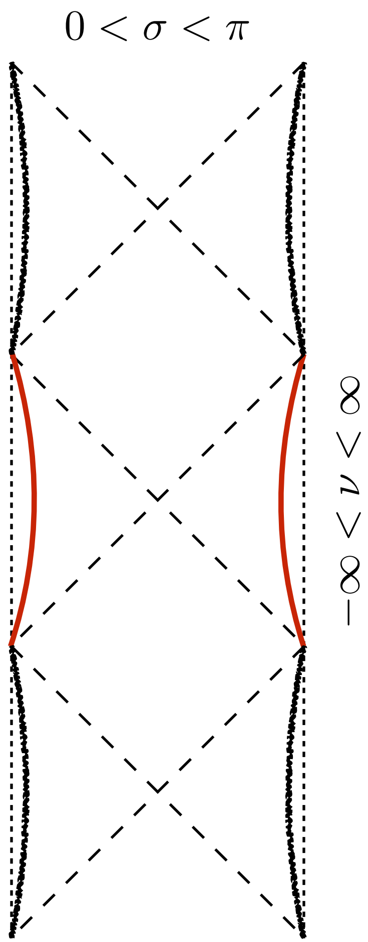
Jackiw-Teitelboim black hole

$$ds^2 = \frac{L^2}{\sin^2 \sigma}\left(-\, d\nu^2 + d\sigma^2\right) \qquad \varphi(\nu,\sigma) = \varphi_H \, \frac{\cos\nu}{\sin\sigma}$$

Boundary curve: $\varphi = \varphi_B \ll \varphi_0$

$\Rightarrow \quad \sin\sigma = \varepsilon \, \cos\nu \quad$ with $\quad \varepsilon \equiv \dfrac{\varphi_H}{\varphi_B} \ll 1$

JT singularity: $\varphi + \varphi_0 = 0$



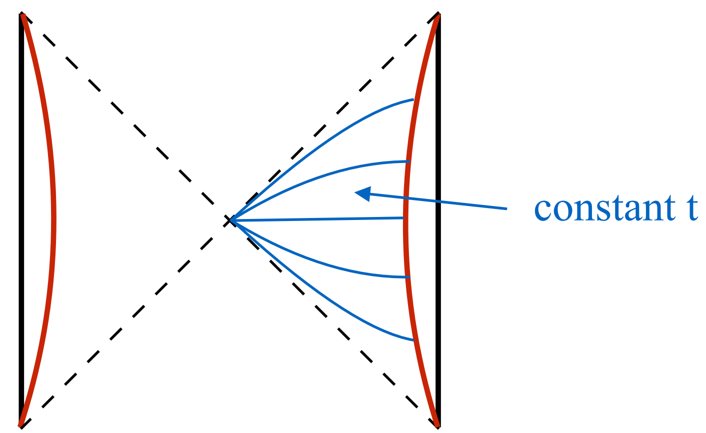
JT black hole in "Schwarzschild" coordinates

$$ds^2 = -\frac{r^2 - r_H^2}{L^2}\, dt^2 + \frac{L^2}{r^2 - r_H^2}\, dr^2 \qquad\qquad \varphi = \varphi_H \,\frac{r}{r_H}$$

AdS₂ scaling: $r \to \lambda r\,, \quad t \to \lambda^{-1} t$

Event horizon: $r = r_H$

BH temperature: $T = \dfrac{r_H}{2\pi L^2}$



Relation between global time and Schwarzschild time (at boundary):

$$\tan\left(\frac{\nu}{2} + \frac{\pi}{4}\right) = e^{2\pi T\, t} + O(\varepsilon^2) \quad \text{at} \quad r = r_B$$

$$\nu = 0 \;\Leftrightarrow\; t = 0$$

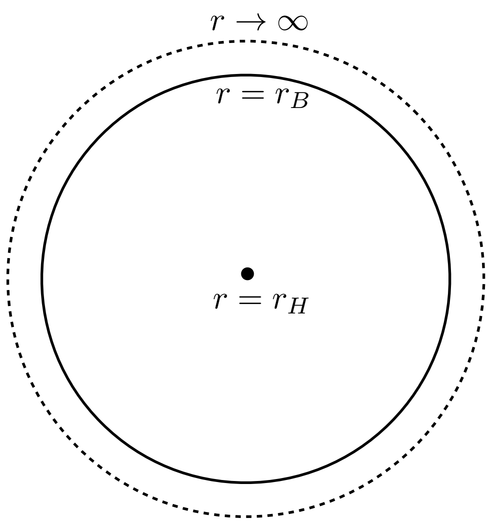
$$\nu \to \pm\frac{\pi}{2} \;\Leftrightarrow\; t \to \pm\infty$$

$$\frac{d\nu}{dt} \approx 4\pi T\, e^{-2\pi T\, t} \quad \text{as} \quad t \to \infty$$

JT black hole thermodynamics

$$ds^2 = \frac{r^2 - r_H^2}{L^2}\, d\tau^2 + \frac{L^2}{r^2 - r_H^2}\, dr^2$$

$$\varphi = \varphi_H\, \frac{r}{r_H}$$



On-shell Euclidean action:

$$S_E = \beta\, F = -S + \beta\, E$$

zero temperature entropy $S_0 = 2\pi\varphi_0$

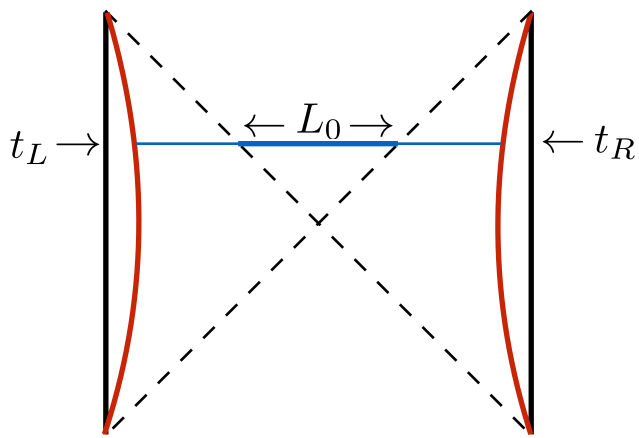
BH entropy: $S = 2\pi\varphi_0 + 4\pi^2 L^2 \frac{\varphi_B}{r_B} T$

BH mass: $E = 2\pi^2 L^2 \frac{\varphi_B}{r_B} T^2$

$$\frac{dE}{dT} = T\, \frac{dS}{dT}$$

Almheiri, Polchinski (2014)
Maldacena, Stanford, Yang (2016)
…..

"$C = V$" for JT black hole



Consider geodesic connecting t_L and t_R on left and right boundaries and calculate the geodesic length L_0 inside BH.

"volume" of maximal slice: $V(t_L, t_R) \sim (G_N \varphi_0)\, L_0$

transverse area

Complexity: $\mathcal{C} \sim \dfrac{V}{G_N L} \sim \dfrac{\varphi_0\, L_0}{L}$

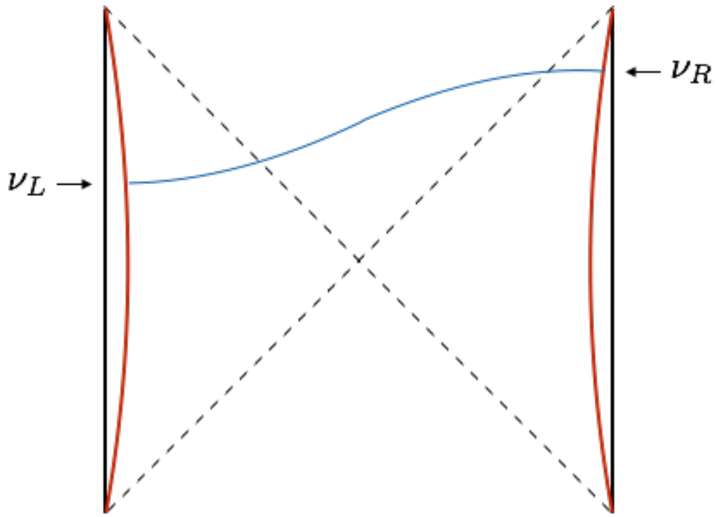
Calculation simplifies for $t_L = t_R = t$

$\tan\left(\frac{\nu}{2} + \frac{\pi}{4}\right) \approx e^{2\pi T t}$

$$L_0 = L \int_{\frac{\pi}{2}-\nu}^{\frac{\pi}{2}+\nu} \frac{d\sigma}{\sin\sigma} = 2L \log\left[\tan\left(\frac{\pi}{4} + \frac{\nu}{2}\right)\right] \approx 4\pi L T\, t \quad \text{as} \quad t \rightarrow \infty$$

Now use $S = 2\pi\varphi_0 + O(T) \longrightarrow \dfrac{d\mathcal{C}}{dt} \sim S\, T$ at late times (up to $O(T^2)$ terms)

"$C = V$" on a generic slice



Geodesic connecting t_L and t_R

$$\sin(\nu - \nu_0) = a \, \cos\sigma$$

$$\nu_0 = \frac{\nu_L + \nu_R}{2}\,, \qquad a = \sin\left(\frac{\nu_L - \nu_R}{2}\right)$$

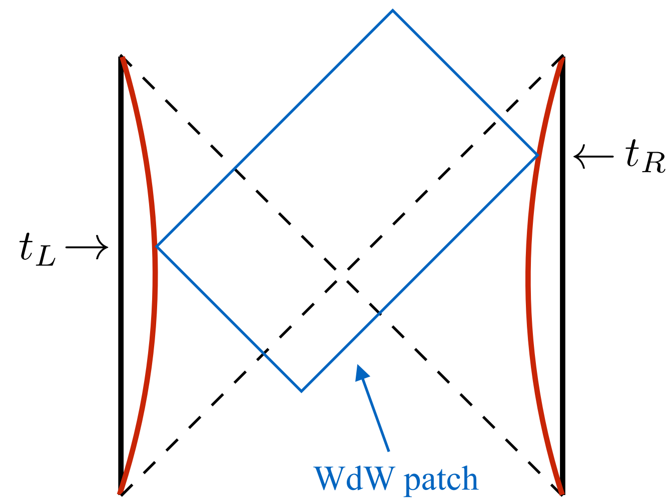
$$L_0 = -L\log\big(\cos\nu_L\big) - L\log\big(\cos\nu_R\big) + 2L\log\Big(\cos\big(\frac{\nu_R - \nu_L}{2}\big)\Big)$$

Complexity $\quad \mathcal{C} \sim \dfrac{S\,L_0}{L} \quad \Rightarrow \quad \dfrac{d\mathcal{C}}{dt_{L,R}} \sim S\,T \quad \text{as} \quad t_{L,R} \to \infty$

“$C = A$” for JT black hole

Complexity = action conjecture $\quad \mathcal{C} = \dfrac{\mathcal{A}_{\text{WdW}}}{\pi}$

Wheeler-DeWitt patch = bulk domain of dependence of a bulk Cauchy slice anchored at boundary



$$S_{JT} = \frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{-g}\,\varphi\left(R + \frac{2}{L^2}\right) + \int_{\partial\mathcal{M}} d\tau\,\varphi\left(K + \frac{1}{L}\right)$$
$$+\,\varphi_0\left(\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{-g}\,R + \int_{\partial\mathcal{M}} d\tau\,K\right)$$

K is ill-defined on null boundaries of WdW patch

$\longrightarrow$ adapt prescription of Lehner et al. arXiv:1609.00207 to case at hand

Appendix C: Action User's Manual

We include a summary of how to evaluate the gravitational action with all its relevant contributions. We write the gravitational action as

$$S_{\mathcal{V}} := \int_{\mathcal{V}} (R - 2\Lambda)\sqrt{-g}\,dV$$
$$+2\,\Sigma_{T_i}\int_{\partial\mathcal{V}_{T_i}} K\,d\Sigma + 2\,\Sigma_{S_i}\text{sign}(S_i)\int_{\partial\mathcal{V}_{S_i}} K\,d\Sigma - 2\,\Sigma_{N_i}\text{sign}(N_i)\int_{\partial\mathcal{V}_{N_i}} \kappa\,dS d\lambda$$
$$+2\,\Sigma_{j_i}\text{sign}(j_i)\oint \eta_{j_i}\,dS + 2\,\Sigma_{m_i}\text{sign}(m_i)\oint a_{m_i}\,dS \qquad \text{(C1)}$$

null boundaries

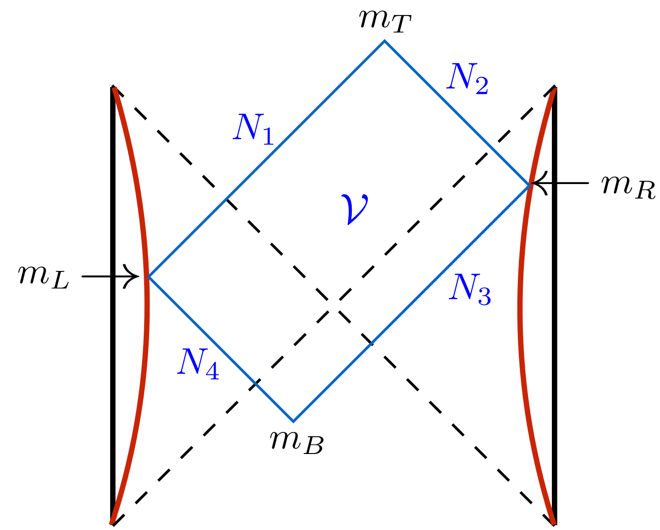
$k^\beta\nabla_\beta k^\alpha = \kappa\,k^\alpha$

$\Rightarrow \kappa = 0$ if λ is affine

“$C = A$” for JT black hole - p.2

Geodesic null boundaries: $(k_1^\nu, k_1^\sigma) = \frac{1}{\frac{d\nu_L}{dt_L}}(\sin^2\sigma, \sin^2\sigma)$ etc.

Normalization condition: $k \cdot \frac{\partial}{\partial t} = -L$ on boundary



$$\longrightarrow \quad \mathcal{A} = \frac{\varphi_0}{2}\int_{\mathcal{V}} d^2x\sqrt{-g}R + \varphi_0 \sum_i \mathrm{sign}(m_i)\, a_{m_i}$$
$$+ \frac{1}{2}\int_{\mathcal{V}} d^2x\sqrt{-g}\,\varphi\left(R + \frac{2}{L^2}\right) + \sum_i \varphi_i\, \mathrm{sign}(m_i)\, a_{m_i}$$

i) topological terms: bulk term cancels against corner contributions

ii) remaining bulk term vanishes on shell

iii) left and right corner terms do not depend on time

iv) top and bottom corner terms grow linearly with time but they cancel at late times

$$\frac{d\mathcal{A}}{dt_{L,R}} \to 0 \quad \text{as} \quad t_{L,R} \to \infty$$

The action on the WdW patch does not grow at late times!

Does this mean that “$C = A$” fails? No, but we need to remember how the JT theory arises in the context of higher-dimensional charged BH’s

3+1-dimensional charged BH

Our starting point is the 3+1-dimensional Einstein-Maxwell theory with action

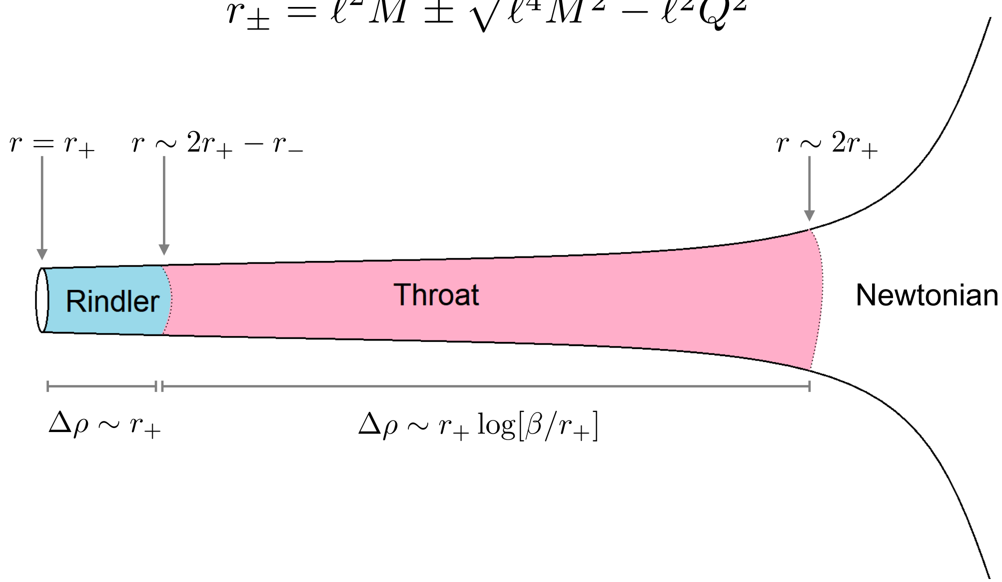
$$\mathcal{S} = \frac{1}{16\pi}\int_{\mathcal{M}} d^4x\,\sqrt{-G}\,\left(\frac{1}{\ell^2}R - F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{8\pi\ell^2}\int_{\partial\mathcal{M}} d^3y\,\sqrt{-h}\,(K - K_0)\,,$$

where $\ell = \sqrt{G_N}$ is the 3+1-dimensional Planck length

Reissner-Nordström black hole with electric charge $Q > 0$ and mass $M \geq Q/\ell$

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\,, \\ f(r) &= \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right), \\ F_{rt} &= \frac{Q}{r^2}\,, \end{aligned}$$

$$r_\pm = \ell^2 M \pm \sqrt{\ell^4 M^2 - \ell^2 Q^2}$$



JT model from spherical reduction

Navarro-Salas, Navarro (1999)

Spherically symmetric ansatz: $$ds^2 = \frac{1}{\sqrt{2\Phi}} g_{\alpha\beta}\, dx^\alpha dx^\beta + 2\ell^2 \Phi\, d\Omega^2$$

1+1 D metric

transverse area is a scalar field in 1+1 D

Inserting into original action gives 1+1 D action of an Einstein-Maxwell-Dilaton theory

$$\mathcal{S}_{2d} = \frac{1}{2} \int d^2x \sqrt{-g} \big(\Phi R + \frac{1}{\ell^2} (2\Phi)^{-\frac{1}{2}} - \frac{\ell^2}{2} (2\Phi)^{\frac{3}{2}} F_{\alpha\beta} F^{\alpha\beta} \big) + \int dy^0 \sqrt{-\gamma_{00}} \big(\Phi K - \frac{1}{\ell} (2\Phi)^{\frac{1}{4}} \big)$$

The field equations of the 1+1-dimensional theory are,

$$\begin{aligned}
0 &= \nabla_\alpha \left(\Phi^{3/2} F^{\alpha\beta} \right), \\
0 &= R - \frac{1}{\ell^2} (2\Phi)^{-3/2} - \frac{3}{2} \ell^2 (2\Phi)^{1/2} F^2 , \\
0 &= \nabla_\alpha \nabla_\beta \Phi - g_{\alpha\beta} \Big(\nabla^2 \Phi - \frac{1}{2\ell^2} (2\Phi)^{-1/2} \Big) + \ell^2 (2\Phi)^{3/2} \Big(F_{\alpha\gamma} F_\beta{}^\gamma - \frac{1}{4} g_{\alpha\beta} F^2 \Big)
\end{aligned}$$

The Maxwell equation determines the electromagnetic field strength in terms of the dilaton

$$F_{\alpha\beta} = \frac{Q}{\ell^2} (2\Phi)^{-3/2}\, \varepsilon_{\alpha\beta}$$

and this can be used to eliminate the gauge field from the remaining equations

Spherical reduction (p.2)

Remaining field equations for the metric and dilaton

$$
\begin{aligned}
0 &= R - \frac{1}{\ell^2}(2\Phi)^{-3/2} + \frac{3Q^2}{\ell^2}(2\Phi)^{-5/2}\,, \\
0 &= \nabla_\alpha \nabla_\beta \Phi - g_{\alpha\beta}\Big(\nabla^2\Phi - \frac{1}{2\ell^2}(2\Phi)^{-1/2} + \frac{Q^2}{2\ell^2}(2\Phi)^{-3/2}\Big)
\end{aligned}
$$

Now expand the dilaton around its value at the horizon of an extremal RN black hole: $\Phi = \frac{Q^2}{2} + \varphi$

and work order by order in φ/Q^2

$$
\longrightarrow \qquad
\begin{aligned}
0 &= R + \frac{2}{L^2}\,, \\
0 &= \nabla_\alpha \nabla_\beta \varphi - g_{\alpha\beta}\Big(\nabla^2\varphi - \frac{1}{L^2}\,\varphi\Big)
\end{aligned}
$$

JT equations with $L = Q^{3/2}\,\ell$

Q: Can the JT action be obtained by integrating out the gauge field and considering the near-horizon limit?

A: Yes, but there is a twist.

Eliminating the gauge field from the 1+1 action, as it stands, leads to a dilaton gravity theory but one with a wrong-sign effective potential for the dilaton.

This kind of sign flip occurs any time a dynamical variable carrying kinetic energy is integrated out in favor of a potential energy term.

The problem is solved by adding an EM boundary term to the original action.

Electromagnetic boundary terms

Our 3+1 D action did not have any boundary terms for the Maxwell field and A_μ is kept fixed at the boundary.

In the Euclidean formalism this corresponds to a thermal ensemble at fixed chemical potential where the total electric charge of the system is allowed to fluctuate.

$$\mathcal{S}_E = \beta F\big|_\mu = -S + \beta M - \beta\mu Q$$

If we add the following boundary term to the action

$$\mathcal{S}_b^{\text{em}} = \frac{1}{4\pi}\int_{\partial\mathcal{M}} d^3y\,\sqrt{-h}\,\hat{n}_\mu\,F^{\mu\nu}A_\nu$$

then free variations of A_μ at the boundary are allowed and the corresponding thermal ensemble is that of fixed charge but varying chemical potential

$$\beta F\big|_Q = -S + \beta M$$

with $\quad S = \pi Q^2 + 4\pi^2 Q^3 \ell T \quad$ and $\quad M = \dfrac{Q}{\ell} + 2\pi^2 Q^3 \ell T^2$

Comparing expressions for JT black hole: $\quad S = 2\pi\varphi_0 + 4\pi^2 L^2 \dfrac{\varphi_B}{r_B} T \qquad M_{\text{2d}} = 2\pi^2 L^2 \dfrac{\varphi_B}{r_B} T^2$

$$\longrightarrow \qquad \varphi_0 = \frac{Q^2}{2} \qquad \text{and} \qquad \varphi = \frac{r}{\ell}$$

(1) JT model describes RN black holes at fixed Q

(2) Higher dimensional embedding provides a reference scale

Electromagnetic boundary terms (p.2)

If the electromagnetic boundary term is included in the 3+1D action, then the 1+1 D action will include its spherical reduction

$$\mathcal{S}^{\rm em}_{b,2d} = \ell^2 \int dy^0 \sqrt{-\gamma_{00}}\,(2\Phi)^{\frac{3}{2}}\,\hat{n}_\alpha\, F^{\alpha\beta} A_\beta$$

Adding a boundary term involving the gauge field does not change its dynamical equations, i.e. the Maxwell equations are not affected, but the boundary term contributes to the effective dilaton potential that results from integrating out the gauge field

Write the boundary term as a 1+1-dimensional bulk term involving a total derivative,

$$\begin{aligned}\mathcal{S}^{\rm em}_{b,2d} &= \ell^2 \int d^2x \sqrt{-g}\,\nabla_\alpha\big((2\Phi)^{\frac{3}{2}}\,F^{\alpha\beta} A_\beta\big) \\ &= \frac{\ell^2}{2} \int d^2x \sqrt{-g}(2\Phi)^{\frac{3}{2}}\,F^{\alpha\beta} F_{\alpha\beta}\,.\end{aligned}$$

This has the same form as the electromagnetic bulk term but with a coefficient in front that is twice as large and of opposite sign

$$\longrightarrow \qquad \mathcal{S} = \frac{1}{2}\int d^2x\sqrt{-g}\Big(\Phi R + \frac{1}{\ell^2}(2\Phi)^{-\frac{1}{2}} - \frac{Q^2}{\ell^2}(2\Phi)^{-\frac{3}{2}}\Big)$$

Now write $\Phi = \dfrac{Q^2}{2} + \varphi_0$ and work order by order in φ

JT theory

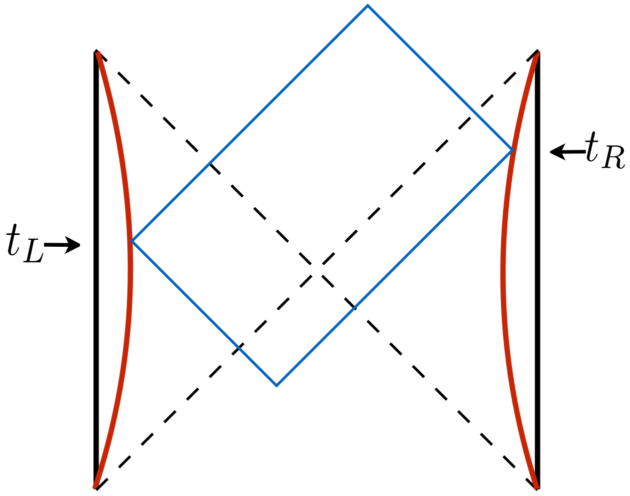
$$\longrightarrow \qquad \mathcal{S} = \frac{Q^2}{4}\int d^2x\sqrt{-g}\,R + \frac{1}{2}\int d^2x\sqrt{-g}\,\varphi\,\Big(R + \frac{2}{L^2}\Big) + \ldots$$

"$C = A$" revisited

The improved WdW patch action for "$C = A$" calculation gives a finite growth rate at late times

$$\mathcal{A} = \mathcal{A}_{JT} + \frac{Q^2}{\ell^2}\int d^2x\sqrt{-g}\,(2\Phi)^{-3/2}$$
$$\approx \mathcal{A}_{JT} + \frac{1}{Q\ell^2}\int d^2x\sqrt{-g}$$
$$= -2Q^2\log(\cos\nu_R) - 2Q^2\log(\cos\nu_L) + \dots$$

$$\longrightarrow \quad \frac{d\mathcal{A}}{dt_{L,R}} = 4\,S\,T + O(T^2) \quad \text{as} \quad t_{L,R} \to \infty$$



Momentum - operator size correspondence

Susskind (2018)

Consider a black hole whose holographic dual is perturbed by a simple operator W.
In the boundary theory, the perturbation then grows with time, *i.e.*

$$W(t) \equiv U(t)^{\dagger}\, W\, U(t)$$

becomes an increasingly complicated operator.

If the scrambling dynamics is generated by a q-local Hamiltonian (with finite q) then the size of $W(t)$ will grow exponentially for some period, with a universal exponent.

Maldacena, Shenker, Stanford (2015); Roberts, Stanford, Streicher (2018); ….

size of a generic k-local operator: $s(k) \sim O(k)$

In the bulk, the perturbation W creates a particle wave-packet that then falls inwards.
As it falls towards the black hole, the particle accelerates.

Consider a 3+1 D Schwarzschild black hole and assume the initial size of W is small:

$$s(0) = 1$$

Size/momentum conjecture: the size of the operator is dual to the radial momentum of the infalling particle (as measured by a static observer at fixed radius)

$$\boxed{s(t) \sim R_s |P(t)|}$$

In Rindler region ($r < 2R_s$) the momentum of infalling particle grows exponentially with Rindler time (with a universal exponent)

$$P(\tau) \sim E_0 e^{\tau}$$

Susskind (2018)

Maximal operator size and BH scrambling time

The size of an operator cannot exceed the entropy S of the black hole $s(t) \leq S$

The operator *scrambling time* is the time it takes to saturate this maximum size $s(t_*) = S$

If $s(0) = s_i$ then $s_i \exp\left(\frac{2\pi t_*}{\beta}\right) = S$ and $t_* = \frac{\beta}{2\pi} \log\left(S/s_i\right)$

Maldacena, Shenker, Stanford (2015)

The initial size equals the added entropy due the initial perturbation

$$s_i = \delta S = \frac{\delta E}{T} \qquad \longrightarrow \qquad t_* = \frac{\beta}{2\pi} \log \frac{S}{\delta S}$$

If the particle created in the bulk has initial energy $\delta E_0 \sim \frac{1}{R_s}$ then $s_i \sim 1$

Scrambling appears to work differently for charged black holes.

extremal black hole entropy

$$t_* = \frac{\beta}{2\pi} \log\left(\frac{S - S_0}{\delta S}\right)$$

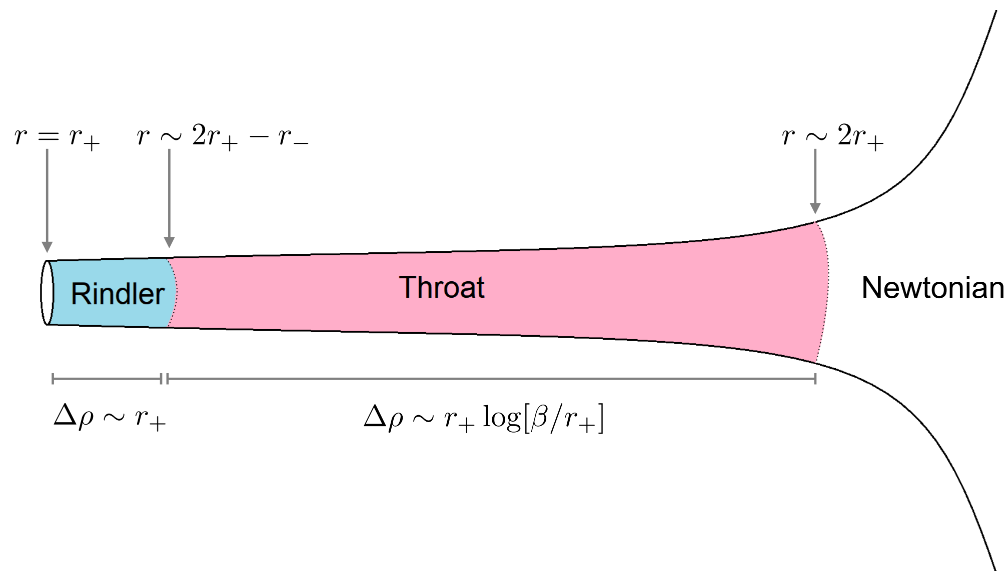
Leichenauer (2014)

Are the extremal degrees of freedom somehow decoupled, leaving only the non-extremal component to actively scramble?

Falling into a charged BH

Reissner-Nordström black hole with electric charge $Q > 0$ and mass $M \geq Q/\ell$

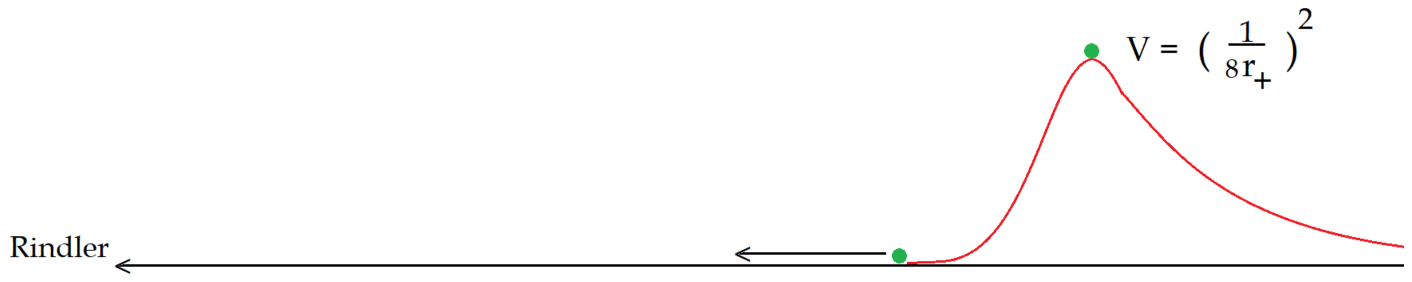
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 ,$$
$$f(r) = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right) ,$$
$$F_{rt} = \frac{Q}{r^2} ,$$
$$r_\pm = \ell^2 M \pm \sqrt{\ell^4 M^2 - \ell^2 Q^2}$$



Hawking temperature: $T = \frac{1}{4\pi}\left(\frac{r_+ - r_-}{r_+^2}\right)$

Dimensionless parameter: $\frac{\beta}{r_+} = \frac{4\pi r_+}{r_+ - r_-} \gg 1 \quad \text{at low } T$

Radial momentum in RN background



Potential barrier at outer edge of RN throat

Particle created at top of potential barrier gains momentum falling into BH

Refined size/momentum conjecture: the size of the operator is dual to the radial momentum of infalling particle (as measured by fiducial observer) in units of local energy scale

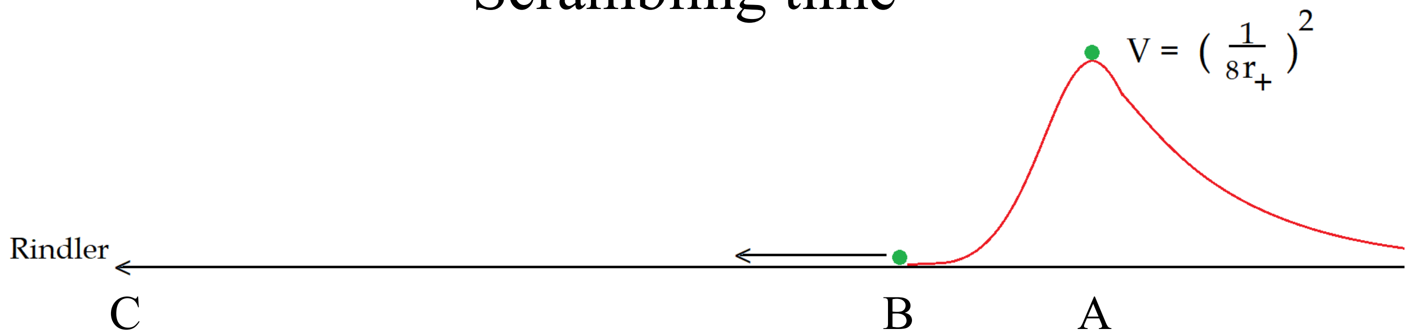
Brown et al. (2018)

$$s(t) \sim \tilde{\beta}(r)\,|P(t)|$$

$\tilde{\beta}(r)$ is the inverse temperature of a black hole with the same charge (but larger mass) that has its event horizon at r

Reduces to Susskind's original conjecture for Schwarzschild BH

Scrambling time



A: Particle created at top of potential barrier and gains momentum falling into BH

B: Particle enters throat region: $P \approx 1/r_+ \,, \qquad \tilde{\beta} \approx r_+$

The particle gains momentum falling: $P \approx 1/r_+ + t/r_+^2 \,, \qquad \tilde{\beta} \approx t + r_+$

Size of dual operator grows as particle traverses BH throat: $s(t) \sim P\,\tilde{\beta} \approx \dfrac{(t+r_+)^2}{r_+^2}$

C: Particle enters Rindler region after a time of order β: $s(\beta) \sim \dfrac{\beta^2}{r_+^2}$

Scrambling time:

$$\left.\begin{array}{ll} \dfrac{\beta^2}{r_+^2} e^{\tau_*} = S & \tau_* = \log\left(S\dfrac{r_+^2}{\beta^2}\right) \\[2ex] \delta S = \dfrac{\beta}{r_+} & S - S_0 \approx \dfrac{r_+}{\beta} S \end{array}\right\} \longrightarrow \quad \tau_* = \log \frac{(S-S_0)}{\delta S}$$

Operator size in SYK model $s(t) = 1 + \left\{ \dfrac{J\beta}{2\pi} \sinh\left(\dfrac{2\pi t}{\beta}\right)\right\}^2$

X.L. Qi, A. Streicher - arXiv:1810.11958

Conclusions

- Size = momentum conjecture can explain the parametrically short scrambling time that is found for near-extremal RN - black holes
- Both "C = V" and "C = A" give expected results for near-AdS_2 BH's

— but not all actions are equal

Thank you!