κ-deformed BMS symmetry

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Overview

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Hopf algebras

- Bialgebra $(\mathcal{A}, m, \Delta, \eta, \epsilon) \equiv$ vector space with algebra and coalgebra structure
- Compatibility between these structures: Δ is algebra homomorphism or m is coalgebra homomorphism

$$\Delta \circ m = m_{\mathcal{A}\otimes\mathcal{A}} \circ (\Delta \otimes \Delta) = (m \otimes m) \circ \Delta_{\mathcal{A}\otimes\mathcal{A}} \tag{1}$$

$$\epsilon \circ m = m_{\mathbb{K}} \circ (\epsilon \otimes \epsilon) \tag{2}$$

- Hopf algebra $\mathcal{A} \equiv$ bialgebra with antipode

$$m \circ (S \otimes \mathrm{id}) \circ \Delta = \eta \circ \epsilon = m \circ (\mathrm{id} \otimes S) \circ \Delta \tag{3}$$

Hopf algebras

- Primitive elements

$$\Delta(x) = \mathbb{1} \otimes x + x \otimes \mathbb{1} \Rightarrow S(x) = -x \tag{4}$$

- Grouplike elements

$$\Delta(x) = x \otimes x \Rightarrow S(x) = x^{-1} \tag{5}$$

- Dual pairing between Hopf algebras $\mathcal{A}, \mathcal{B}$

$$\langle a_1 a_2, b \rangle = \langle a_1 \otimes a_2, \Delta(b) \rangle \tag{6}$$

Noncommutativity of spacetime

- Consider Hopf algebra of momenta $\mathcal{P}$, dual to spacetime Hopf algebra $\tilde{\mathcal{P}}$

$$\langle P_\mu, x^\nu \rangle = \delta_\mu^\nu \tag{7}$$

- Induces action from $\mathcal{P}$ on $\tilde{\mathcal{P}}$

$$P_\mu \triangleright x^\nu = \langle P^\mu, x^\nu_{(1)} \rangle \, x^\nu_{(2)} \tag{8}$$

$$\Delta(x^\nu) = x^\nu_{(1)} \otimes x^\nu_{(2)} = x^\nu \otimes \mathbb{1} + \mathbb{1} \otimes x^\nu \tag{9}$$

Noncommutativity of spacetime

- Algebra module:

$$P_\mu \triangleright (x^\nu x^\lambda) = (P_{\mu_{(1)}} \triangleright x^\nu)(P_{\mu_{(2)}} \triangleright x^\lambda) \tag{10}$$

- If coproduct of $\mathcal{P}$ is not primitive/symmetric:

$$\Rightarrow P_\mu \triangleright [x^\nu, x^\lambda] \sim \frac{1}{\kappa} \neq 0 \tag{11}$$

Noncommutativity of spacetime

- Non-trivial coalgebra structure corresponds to non-commutativity
- κ observer independent, dimension of mass
- Indicates relation to QG, also motivated by string theory/ LQG
- Basis of noncommutative field theory

Non additive momenta

- Consider representation (algebra module) of Hopf algebra $\mathcal{P}$ on two particle Fock space

$$P_\mu^{\mathcal{H}^2} \triangleright |p^1\rangle \otimes |p^2\rangle = \Delta(P_\mu^{\mathcal{H}}) \triangleright |p^1\rangle \otimes |p^2\rangle \tag{12}$$

$$P_\mu^{\mathcal{H}} |p^1\rangle = p_\mu^1 |p^1\rangle \tag{13}$$

- Primitive coproduct $\Rightarrow$ Leibniz rule, otherwise momenta not additive
- In particular momenta not linearly separable

κ Poincaré algebra

- Deformation of Poincaré algebra with primitive coproduct
- "Minimal" deformation, i.e. assosciativity preserved
- time-like, space-like and light-like κ Poincaré

$$[x^\mu, x^\nu] = \frac{i}{\kappa}(\tau^\mu x^\nu - \tau^\nu x^\mu) \tag{14}$$

- Light-like deformation can be obtained by twist
- Light-cone coordinates

$$x^+ = x^0 + x^3, \quad x^- = x^0 - x^3 \tag{15}$$

κ Poincaré

$$[M_{\mu\nu}, M_{\rho\lambda}] = i(g_{\mu\lambda}M_{\nu\rho} - g_{\nu\lambda}M_{\mu\rho} + g_{\nu\rho}M_{\mu\lambda} - g_{\mu\rho}M_{\nu\lambda}), \tag{16}$$

$$[M_{\mu\nu}, P_\rho] = i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu), \quad [P_\mu, P_\nu] = 0 \tag{17}$$

$$\Delta(P_\mu) = P_\mu \otimes \Pi_\tau + \mathbb{1} \otimes P_\mu - \frac{\tau_\mu}{\kappa} P^\alpha \Pi_\tau^{-1} \otimes P_\alpha - \frac{\tau_\mu}{2\kappa^2} C_\tau \Pi_\tau^{-1} \otimes P_\tau \tag{18}$$

$$\begin{aligned} \Delta(M_{\mu\nu}) =& M_{\mu\nu} \otimes \mathbb{1} + \mathbb{1} \otimes M_{\mu\nu} + \frac{1}{\kappa} P^\alpha \Pi_\tau^{-1} \otimes (\tau_\nu M_{\alpha\mu} - \tau_\mu M_{\alpha\nu}) \\ & - \frac{1}{2\kappa^2} C_\tau \Pi_\tau^{-1} \otimes (\tau_\mu M_{\tau\nu} - \tau_\nu M_{\tau\mu}) \end{aligned} \tag{19}$$

BMS algebra

- Symmetry of flat spactime: Poincaré algebra
- What is the symmetry of asymptotically flat spacetime?

$$ds^2 = -du^2 - dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \tag{20}$$

$$ds^2 = -Udu^2 - 2e^{2\beta}dudr + g_{AB}\left(d\Theta^A + \frac{1}{2}U^A du\right)\left(d\Theta^B + \frac{1}{2}U^B du\right) \tag{21}$$

$$\Theta^A = (z, \bar{z}) \tag{22}$$

BMS algebra

- Killing vectors of the asymptotically flat spacetime:

$$\xi_{T,R} = \left(T + \frac{u}{2} D^A R^A\right) \partial_u + \left(\left(1 + \frac{u}{2r}\right) R^A - \frac{u}{2r} D^A D_B R^B - \frac{1}{r} D^A T\right) \partial_A + \left(-\frac{r+u}{2} D_A R^A + \frac{1}{2} D_A D^A T\right) \partial_r \tag{23}$$

- Lie derivative of vector fields defines Lie bracket
- Parametrize functions:

$$R_n^z \equiv l_n = -z^{n+1}, \quad R_n^{\bar{z}} \equiv \bar{l}_n = -\bar{z}^{n+1}, \quad T_{pq} \equiv \frac{z^p \bar{z}^q}{1 + z\bar{z}} \tag{24}$$

$$[l_m, l_n] = (m-n)l_{m+n}, \quad [\bar{l}_m, \bar{l}_n] = (m-n)\bar{l}_{m+n}, \quad [l_m, \bar{l}_n] = 0 \tag{25}$$

$$[l_l, T_{m,n}] = \left(\frac{l+1}{2} - m\right) T_{m+l,n}, \quad [\bar{l}_l, T_{m,n}] = \left(\frac{l+1}{2} - n\right) T_{m,n+l} \tag{26}$$

- T_{pq} supertranslations - related to gravitational memory effect
- $l_n, \bar{l}_n$ superrotations - related to spin memory effect

BMS algebra

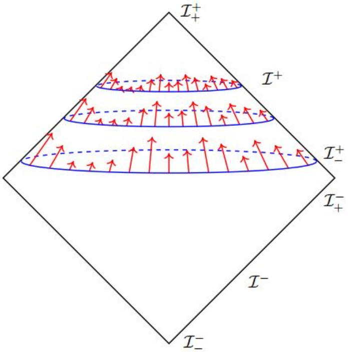

Figure: Action of a supertranslation that shifts retarded time u indepently at every angle.[Strominger, 2018]

Deformation by twisting

- Is there a straightforward way to obtain a Hopf algebra from a Lie algebra $\mathcal{A}$?
- Consider universal enveloping algebra $U(\mathcal{A})$ with

$$\Delta_0(x) = \mathbb{1} \otimes x + x \otimes \mathbb{1}, \quad S_0(x) = -x, \quad \epsilon(x) = 0 \tag{27}$$

- Deform this trivial structure but keep assosciativity

Deformation by twisting

- Have to find invertible twisting element $\mathcal{F}$ satisfying

$$\mathcal{F}_{12} \cdot (\Delta_0 \otimes \mathbb{1})(\mathcal{F}) = \mathcal{F}_{23} \cdot (\mathbb{1} \otimes \Delta_0)(\mathcal{F}), \quad (\epsilon \otimes \mathrm{id})\mathcal{F} = \mathbb{1} \tag{28}$$

- If r-matrix exists this is guaranteed

$$\mathcal{F} = \exp\left(-i\frac{1}{\kappa}M_{+i} \otimes P^i\right)\exp\left(-iM_{+-} \otimes \log\left(1 + \frac{P_+}{\kappa}\right)\right) \tag{29}$$

$$\Delta(x) = \mathcal{F}\Delta_0(x)\mathcal{F}^{-1}, \quad S(x) = vS_0(x)v^{-1} \tag{30}$$

$$v = m \circ (\mathrm{id} \otimes S_0)\mathcal{F}, \quad v^{-1} = m \circ (S_0 \otimes \mathrm{id})\mathcal{F}^{-1} \tag{31}$$

Results

- Express twisting element in terms of BMS generators

$$k_n = l_n + \bar{l}_n, \quad \bar{k}_n = -i(l_n - \bar{l}_n), \tag{32}$$

$$S_{mn} = \frac{1}{2}(T_{mn} + T_{nm}), \quad A_{mn} = -\frac{i}{2}(T_{mn} - T_{nm}) \tag{33}$$

$$\mathcal{F} = \exp\left(\frac{1}{\sqrt{2\kappa}}(ik_1 \otimes S_{01} + i\bar{k}_1 \otimes A_{01})\right) \exp\left(-k_0 \otimes \log\left(1 + \frac{iS_{11}}{\sqrt{2\kappa}}\right)\right) \tag{34}$$

- Use Hadamar formula:

$$e^A B e^{-A} = \sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{[A, [A, ...[A,}_{\text{n times}} B]..]. \tag{35}$$

Results

$$\Delta(S_{pq}) = \mathbb{1} \otimes S_{pq} + S_{pq} \otimes \left(\mathbb{1} + (p+q-1)\frac{P_+}{\kappa} \right) + \frac{i}{\sqrt{2}\kappa}(1-p)\left(A_{p+1,q} \otimes P_2 + S_{p+1,q} \otimes P_1\right) + \frac{i}{\sqrt{2}\kappa}(1-q)\left(A_{p,q+1} \otimes P_2 + S_{p,q+1} \otimes P_1\right) + \mathcal{O}\left(\frac{1}{\kappa^2}\right) \quad (36)$$

- Poincaré subalgebra forms also a sub-Hopf algebra
- Non-additiv supertransformations/momenta

Black hole information loss paradox

- Black holes evaporate via *thermal* Hawking radiation to *unique* vacuum $\rightarrow$ Information about matter that formed the BH is lost
- Only 10 Poincaré hair + gauge charges
- Recent propossal by Hawking, Perry, Strominger [2]: BHs related by super transformation are physically inequivalent and conservation of all the corresponding "soft" charges could store information of the infalling matter
- Objection by Porrati, Bousso [3]: Soft theorems imply that the time evolution of soft d.o.f. decouples from the hard part $\rightarrow$ no constraints on the outgoing hard modes

Application of κ-BMS

- For each supertranslation

$$S_{mn}^{\text{tot, bh}} = S_{mn}^{\text{tot, e}} + S_{mn}^{\text{tot, l}} \tag{37}$$

$$S_{nm}^{\text{tot}} \left|P_\mu, 0\right\rangle^{(\text{hard})} \otimes \left|0, S_{pq}\right\rangle^{(\text{soft})} = \Delta(S_{nm}) \left|P_\mu, 0\right\rangle^{(\text{hard})} \otimes \left|0, S_{pq}\right\rangle^{(\text{soft})} \tag{38}$$

- Primitive coproduct $\rightarrow$ only Poincaré momentum conservation constrains hard modes
- κ-BMS induces mixing of hard/soft modes $\rightarrow$ objection would be invalid

Outlook

- dual BMS algebra and interpretation
- Local finite dimensionality of the Hopf algebra, q analogue
- better understanding of "tangled soft/hard hair"
- ...

References

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