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Supergravity in a Pencil: From Supergravity to Graphene

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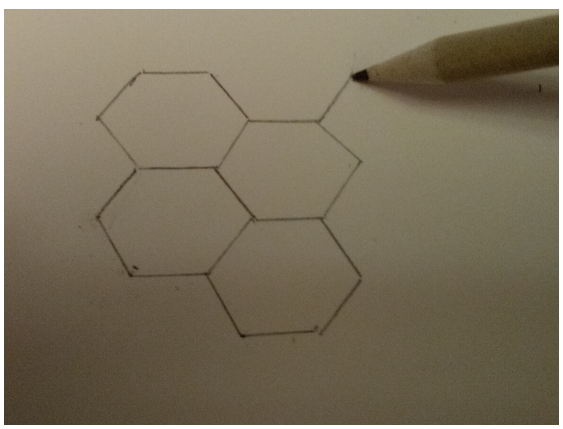
based on JHEP04(2018)007, arXiv:1801.08081, with L. Andrianopoli, R. D'Auria, M. Trigiante
and work in progress with P.A. Grassi, R. Noris, R. Olea, J. Zanelli



MUSEO STORICO DELLA FISICA E CENTRO STUDI E RICERCHE ENRICO FERMI

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Abstract

In the spirit of the holographic correspondence, we have investigated the relation between a $D = 3+1, \mathcal{N} = 2$ AdS_4 supergravity (SUGRA) theory in the presence of a boundary of space-time developed in [L. Andrianopoli, R. D'Auria, JHEP 1408 (2014) 012, arXiv:1405.2010 [hep-th]] and a model (AVZ) presented in [P.D. Alvarez, M. Valenzuela, J. Zanelli, JHEP 1204 (2012) 058, arXiv:1109.3944 [hep-th]] for a charged spin$-\frac{1}{2}$ field in $2+1$ dimensions with OSp(2|2) symmetry, satisfying a Super-Chern-Simons (SCS) Lagrangian, which can describe some condensed matter systems with fermionic excitations in 2+1 dimensions, like graphene. We have found that the constraints on the 3D boundary of $D = 4, \mathcal{N} = 2$ SUGRA can be recovered as equations of motion from a 3D SUGRA with OSp(2|2)$\times$ SO(1,2) invariance. A model where this can be explicitly realized by means of an appropriate choice of the boundary conditions is provided by the "ultraspinning limit" [M.M. Caldarelli, R. Emparan, M.J. Rodriguez, JHEP 0811 (2008) 011, arXiv:0806.1954 [hep-th]] of an AdS_4-Kerr black hole, an asymptotically-AdS_4 solution with an AdS_3 geometry at the boundary. This top-down approach to graphene is more predictive than the bottom-up one, common in the holographic approach to solid state physics.

Plan

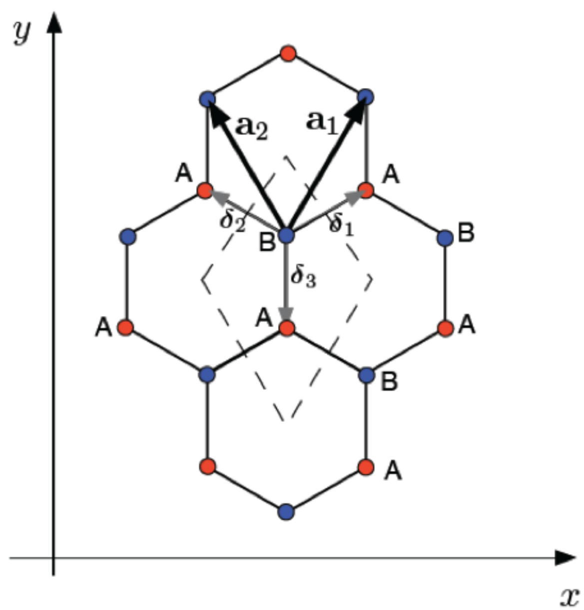
1. Graphene and the Dirac equation
2. Boundary behavior of $\mathcal{N} = 2$, AdS_4 supergravity
3. Explicit $D = 3$ Description: Asymptotic "ultraspinning" limit
4. Comparison with "unconventional" supersymmetry
5. Conclusions and outlook

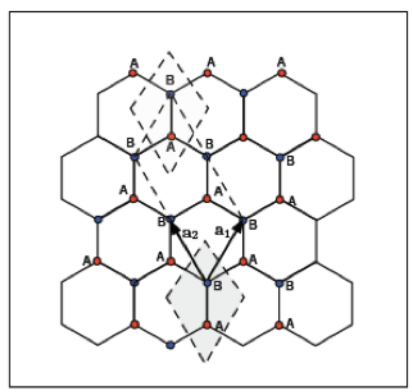
Graphene and the Dirac equation

The graphene honeycomb lattice

Graphene is a two-dimensional layer of carbon atoms (one single layer of graphite).

The carbon atoms in graphene form a honeycomb lattice with a hexagonal structure, due to the sp^2 orbital hybridization.
It is a bipartite lattice composed by two triangular sublattices (sites A and sites B).

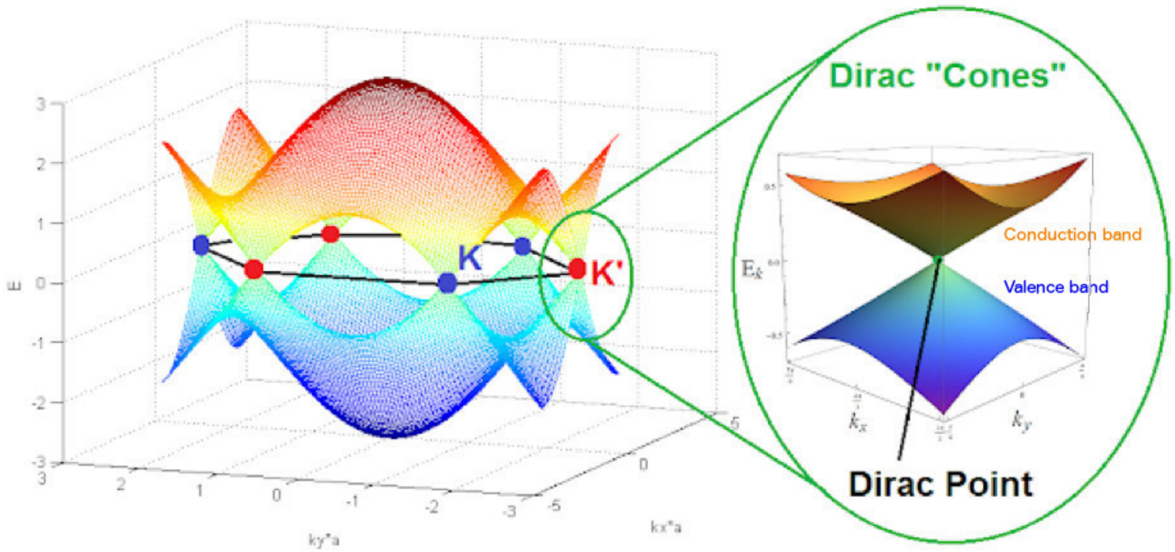




$\Longrightarrow$ Belonging to site A or B defines a spin-like quantum number: Pseudospin, an additional quantum number on top of the actual spin.
$\Longrightarrow$ The spectrum is helical.

The graphene Dirac cone

The Electron Band Structure of graphene



At the Dirac points (for a range of 1eV) the spectrum is linear:

$$\text{Dirac cone:} \qquad E_{\mathbf{k}} = \pm \hbar c |\mathbf{k}|$$

$\Longrightarrow$ Electrons in graphene obey the same type of equations as relativistic Dirac massless particles with

$$c \rightarrow v_F = 10^6 \frac{m}{s} = \frac{c}{300} \text{ Fermi velocity}$$

“Analogue” relativity in condensed matter

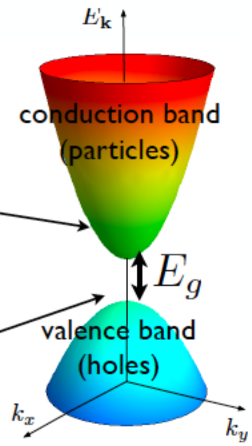
Relativistic energy: $E_{\mathbf{k}} = \pm\sqrt{(\hbar c\mathbf{k})^2 + (mc^2)^2}$

Semiconductors

If m finite

$E_{\mathbf{k}} \simeq E_c + \frac{\hbar^2\mathbf{k}^2}{2m_c^*}$

$E_{\mathbf{k}} \simeq E_v - \frac{\hbar^2\mathbf{k}^2}{2m_v^*}$



bare mass m $\Longrightarrow$ effective mass m^*

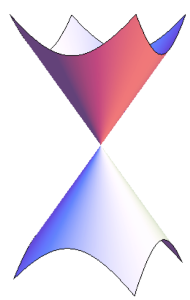
Due to the interaction of the electrons with the lattice atoms, usually $m^* \leq m$. In graphene it actually vanishes.

Graphene

Massless case:

$E_{\mathrm{k}} = \hbar|\mathbf{k}|v_F \rightarrow$

$E_{\mathrm{k}} = -\hbar|\mathbf{k}|v_F \rightarrow$



Light speed c $\Longrightarrow$ Fermi velocity v_F

The powerful methods of the relativistic theories turn out to be a very useful tool for exploring the special properties of graphene possessing a two-dimensional spatially curved surface [A. Cortijo, M. A. H. Vozmediano, Eur. Phys. J. ST 148, 83 (2007), condmat/0612623; Nucl. Phys. B 763, 293 (2007) [Nucl. Phys. B 807, 659 (2009)], cond-mat/0612374], including topologically non trivial cases [M. Cvetič, G.W. Gibbons, Annals Phys. 327 (2012) 2617, arXiv:1202.2938 [hep-th]].

Viceversa, graphene has been proposed as a simple laboratory to check gravitational cosmic phenomena like Hawking-Unruh radiation [A. Iorio, G. Lambiase, Phys. Lett. B 716, 334 (2012), arXiv:1108.2340 [cond-mat.mtrlsci]], or wormholes in bilayer graphene [J. Gonzalez, J. Herrero, Nucl.Phys. B825 (2010) 426-443, arXiv:0909.3057 [cond-mat.mes-hall]].

In the spirit of the gauge/gravity correspondence, which relates a gauge theory in D dimensions to a gravity theory in one dimension higher, we want to relate a Super Chern-Simons Lagrangian yielding the graphene Dirac equation in $D = 2 + 1$ to an AdS_4 supergravity.

Boundary behavior of $\mathcal{N}=2$, AdS$_4$ supergravity

[L. Andrianopoli, R. D'Auria, JHEP 1408 (2014) 012, arXiv:1405.2010]

Denote by: V^a the bosonic vielbein 1–form in superspace;
$\Psi^\alpha_A (A=1,\dots\mathcal{N},\alpha=1,\dots,4)$ the fermionic vielbein 1–form;
$A^{(4)}$ the D=4 gauge field; R^{ab} the curvature;
$\nabla^{(4)}$ the SO$(1,3)$ covariant differential.

Here $a=0,\dots,3$ are $D=4$ flat space-time indices,
$A=1,\dots\mathcal{N}$ labels the $U(\mathcal{N})$ R–symmetry,
$\alpha=1,\dots,4$ is a $D=4$ spinor index, omitted in the following.
Define:

$$R^{ab}\equiv d\omega^{ab}+\omega^a{}_c\wedge\omega^{cb}\,,\qquad \nabla^{(4)}V^a\equiv dV^a+\omega^a{}_b\wedge V^b\,,$$

$$\nabla^{(4)}\Psi_A\equiv d\Psi_A+\frac{1}{4}\omega_{ab}\Gamma^{ab}\wedge\Psi_A-\frac{1}{2\ell}\epsilon_{AB}A^{(4)}\wedge\Psi_B\,.$$

In the presence of a boundary of space-time, local supersymmetry invariance requires appropriate boundary conditions to be imposed:

$$R^{ab}|_{\partial\mathcal{M}}=\left[\frac{1}{\ell^2}V^a\wedge V^b+\frac{1}{2\ell}\bar\Psi_A\Gamma^{ab}\wedge\Psi_A\right]_{\partial\mathcal{M}}\,,\quad \nabla^{(4)}V^a|_{\partial\mathcal{M}}=\frac{i}{2}\bar\Psi_A\Gamma^a\wedge\Psi_A|_{\partial\mathcal{M}}\,,$$

$$dA^{(4)}|_{\partial\mathcal{M}}=\bar\Psi_A\wedge\Psi_B\,\epsilon_{AB}|_{\partial\mathcal{M}}\,,\qquad \nabla^{(4)}\Psi_A|_{\partial\mathcal{M}}=\frac{\mathrm{i}}{2\ell}\Gamma_a\Psi_A\wedge V^a|_{\partial\mathcal{M}}\,,$$

with $\mathcal{M}$ the four-dimensional space-time, $\partial\mathcal{M}$ its three-dimensional boundary, Γ^a the $D=4$ gamma matrices, ℓ the AdS$_4$ radius.

Explicit $D=3$ description: Asymptotic "ultraspinning" limit

Want: local AdS_3 as effective theory on boundary $\partial\mathcal{M}$ at $r\to\infty$.

Fefferman-Graham parametrization: coordinates $x^{\hat\mu}$, $\hat\mu=0,\dots,3$ of $\mathcal{M}$ split into x^μ, $\mu=0,1,2$ on $\partial\mathcal{M}$ + radial coordinate $x^3=r$.
AdS_4 symmetry: $\mathrm{SO}(2,3)\to\mathrm{SO}(1,1)\times\mathrm{SO}(1,2)$

1–forms: $K^i_\pm\equiv\frac{1}{2}\left(V^i\mp\ell\,\omega^{3i}\right)$

Gravitinos: $\Psi_A=\Psi_{+A}+\Psi_{-A}\,,\quad \Gamma^3\Psi_{\pm A}=\pm\mathrm{i}\,\Psi_{\pm A}\,.$

"Ultraspinning limit" of AdS_4-Kerr black-hole [M.M.Caldarelli, R. Emparan, M. J. Rodriguez, JHEP 0811 (2008) 011, arXiv:0806.1954 [hep-th]]:

$$K^i_+(x,r)=\frac{r}{\ell}E^i(x)+\dots\,,\ K^i_-(x,r)=\frac{1}{4}\frac{\ell}{r}E^i(x)+\dots\,,\ V^3(r)=\frac{\ell}{r}dr+\dots\,,$$
$$\Psi_{+A\mu}(x,r)=\sqrt{\frac{r}{\ell}}(\psi_{A\mu},\mathbf{0})+\dots\,,\ \Psi_{-A\mu}(x,r)=\sqrt{\frac{\ell}{r}}\left(\mathbf{0},\frac{\varepsilon}{2}\psi_{A\mu}\right)+\dots\,,$$
$$\omega^{ij}(x,r)=\omega^{ij}(x)+\dots\,,\quad A^{(4)}(x,r)=\varepsilon\,A_\mu(x^\nu)\,dx^\mu+\dots\,.$$

Ellipses: subleading terms in the $r\to\infty$ limit, $\varepsilon=\pm1$,
$\psi_{A\mu}=(\psi_{A\mu\alpha})$, $\alpha=1,2$: $D=2+1$ gravitini (Majorana).

Assume mass of black hole vanishes $\Longrightarrow$ Energy momentum tensor of boundary theory vansihes [A. J. Amsel, G. Compère, Phys. Rev. D **79** (2009) 085006, arXiv:0901.3609 [hep-th]]: Neumann condition.

Manifestly $\mathfrak{osp}(2|2)_{(\varepsilon)} \times \mathfrak{so}(1,2)_{(-\varepsilon)}$ covariant formulation of the boundary theory [A. Achucarro, P. K. Townsend, Phys. Lett. B **229** (1989) 383]

Torsionful spin connections : $\omega^{ij}_{(\pm\varepsilon)} = \omega^{ij} \pm \frac{\varepsilon}{\ell} E_k \epsilon^{ijk} = \epsilon^{ijk}\,\omega_{\pm(\varepsilon)k}$, with covariant derivatives $\mathcal{D}_{(\varepsilon)}$ and curvatures $R^i_{(\pm)}$.

Supersymmetry transformation with parameter ϵ_A:

$$\delta\omega^i_{(-\varepsilon)} = 0\,,\ \delta\omega^i_{(\varepsilon)} = \varepsilon\,\frac{2\mathrm{i}}{\ell}\,\bar{\epsilon}_A\gamma^i\psi_A\,,\ \delta A = 2\epsilon_{AB}\bar{\epsilon}_A\psi_B\,,\ \delta\psi_A = \mathcal{D}_{(\varepsilon)}\epsilon_A - \frac{\varepsilon}{2\ell}\,\epsilon_{AB}\,A\,\epsilon_B \equiv \nabla^{(\varepsilon)}\epsilon_A\,,$$

$\Longrightarrow$ Superalgebra $\mathfrak{osp}(2|2)\times\mathfrak{so}(1,2)$ realized as gauge symmetry.

The boundary conditions imposed by SUSY:

$$\mathcal{R}^i_{(\varepsilon)} = \mathrm{i}\,\frac{\varepsilon}{\ell}\,\bar{\psi}_A\gamma^i\psi_A\,, \mathcal{R}^i_{(-\varepsilon)} = 0\,, \mathcal{D}_{(\varepsilon)}\psi_A = \frac{\varepsilon}{2\ell}\,\epsilon_{AB}\,A\psi_B\,, dA = \epsilon_{AB}\bar{\psi}_A\psi_B$$

can be derived as equations of motions from the Lagrangian:

$$\mathcal{L}^{(3)} = \varepsilon\left(\mathcal{L}_{(\varepsilon)} - \mathcal{L}_{(-\varepsilon)}\right) \equiv \mathcal{L}^{(3)}_{+} - \mathcal{L}^{(3)}_{-} \quad \text{with}$$

$$\mathfrak{osp}(2|2)_{(\varepsilon)}\ \text{SCS} : \mathcal{L}_{(\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(\varepsilon)}d\omega_{(\varepsilon)|i} - \frac{1}{3}\omega^i_{(\varepsilon)}\omega^j_{(\varepsilon)}\omega^k_{(\varepsilon)}\varepsilon_{ijk}\right) + 2\varepsilon\,\bar{\psi}_A\nabla^{(\varepsilon)}\psi_A - \frac{\varepsilon}{2\ell}A\,dA\,,$$

$$\mathfrak{so}(1,2)_{(-\varepsilon)}\ \text{CS} : \mathcal{L}_{(-\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(-\varepsilon)}d\omega_{(-\varepsilon)|i} - \frac{1}{3}\omega^i_{(-\varepsilon)}\omega^j_{(-\varepsilon)}\omega^k_{(-\varepsilon)}\varepsilon_{ijk}\right)\,,$$

$$\nabla^{(\varepsilon)}\psi_A \equiv \left(d + \frac{1}{4}\omega^{ij}_{(\varepsilon)}\gamma_{ij}\right)\psi_A - \frac{\varepsilon}{2\ell}A\,\psi_B\,\epsilon_{AB}\,.$$

Comparison with "unconventional" supersymmetry

[L. Andrianopoli, BLC, R. D'Auria, M. Trigiante, JHEP04(2018)007, arXiv: 1801.08081]

A peculiar feature of the the AVZ model in [Alvarez, Valenzuela, Zanelli, arXiv:1109.3944] is that the spinor 1–form associated with the odd generator of the superalgebra is not a spin–$\frac{3}{2}$ gravitino, but is given in terms a ($\mathcal{N}=2$) spin–$\frac{1}{2}$ field χ_A:

$$\psi_A = \mathrm{i}\, e_i\, \gamma^i \chi_A\,,$$

with e_i a SUSY invariant 1–form space-time dreibein: $\delta e^i = 0$, $\hat{\mathcal{D}} e^i = \frac{1}{\ell}\,\epsilon^{ijk}\, e_j\, e_k\,,$ $\hat{\mathcal{D}}$ covariant derivative of a torsionful conn. ω^i.

In terms of the spinor χ_A, and identifying ω^i with $\omega^i_{(\varepsilon)} = \omega^i_{(-)}$, the SCS Lagrangian of the AVZ model coincides, modulo an overall scaling, with the SCS Lagrangian $\mathcal{L}_{(\varepsilon)}$ *for the choice* $\varepsilon = -1$:

$$\mathcal{L}_{(\varepsilon)} = \frac{\ell}{2}\left(\omega^i_{(\varepsilon)} d\omega_{(\varepsilon)|i} - \frac{1}{3}\omega^i_{(\varepsilon)}\omega^j_{(\varepsilon)}\omega^k_{(\varepsilon)}\varepsilon_{ijk}\right) + 2\varepsilon\, e_i\, \mathcal{D}^{(\varepsilon)} e^i\, \bar{\chi}_A\chi_A - 4\,\mathrm{i}\,\varepsilon\,\bar{\chi}_A \not{\nabla}^{(\varepsilon)}\chi_A\, \mathrm{e}\, d^3x - \frac{\varepsilon}{2\ell} A\, dA\,,$$

where $\mathrm{e} \equiv \det(e_\mu{}^i)$ and $\not{\nabla}^{(\varepsilon)}\chi_A \equiv \gamma^i\,\nabla^{(\varepsilon)}_i \chi_A = \not{\mathcal{D}}^{(\varepsilon)}\chi_A - \frac{\varepsilon}{2\ell}\, A_i \epsilon_{AB}\gamma^i\chi_B\,.$

The equation of motion for χ_A is the Dirac equation:

$$\not{\nabla}^{(\varepsilon)}\chi_A - \frac{\mathrm{i}}{3\ell}\,\varepsilon\chi_A \;=\; 0\,.$$

SUSY transformation of χ_A: $\delta\chi_A = -\frac{\mathrm{i}}{3}\gamma^i\,\nabla^{(\varepsilon)}_i \epsilon_A\,.$

Nieh-Yan-Weyl invariance $\Longrightarrow$ $\bar{\chi}\chi$ free constant of the model.

Some remarks

- In the AVZ model the supersymmetry algebra is realized as a gauge symmetry with fields in the adjoint representation $\Longrightarrow$ The numbers of fermions and bosons do not coincide: "Unconventional supersymmetry" (Nonlinear realization?)
- The spacetime dreibein e^i of the AVZ model does not coincide with the superspace dreibein E^i. Consistency requires:

$$E^i = M(\bar{\chi}\chi)\, e^i \,, \text{ with } M(\bar{\chi}\chi) = \left(1 + \varepsilon\,\frac{\ell}{2}\,\bar{\chi}\chi - \frac{\ell^2}{4}(\bar{\chi}\chi)^2\right).$$

$\Longrightarrow$ The SUSY parameter ϵ_A in $D = 2+1$ is proportional to the propagating spinor field:

$$\epsilon_A = \mathrm{i}N(\bar{\chi}\chi)\,\chi_A \,, \text{ with } N(\bar{\chi}\chi) = \beta\bar{\chi}\chi.$$

- The spinor χ_A is the spin$-\frac{1}{2}$ projection of the $D = 2+1$ gravitino ψ_A: $\chi_A = -\frac{\mathrm{i}}{3}\gamma_i\, e^{i|\mu}\psi_{A\,\mu}$. On the other hand:

$$\psi_{+\,A\,3} = \frac{3\mathrm{i}\varepsilon}{2M(\bar{\chi}\chi)}\left(\frac{\ell}{r}\right)^{\frac{5}{2}}(\chi_A, \mathbf{0})\,, \quad \psi_{-\,A\,3} = \frac{3\mathrm{i}}{M(\bar{\chi}\chi)}\left(\frac{\ell}{r}\right)^{\frac{3}{2}}(\mathbf{0}, \chi_A)\,.$$

$\Longrightarrow$ The spinor χ_A is originating from the radial component of the $D = 3+1$ gravitino field.

Summary: Our results

- In the spirit of the AdS/CFT correspondence, we have recovered from a pure $\mathcal{N}=2$ AdS_4 supergravity with boundary in [Andrianopoli, D'Auria, arXiv:1405.2010] a $D=2+1$ Super-Chern-Simons theory in [Alvarez, Valenzuela, Zanelli, arXiv:1109.3944], describing the behavior of graphene near the Dirac points.
- The AVZ model displays $\mathcal{N}=2$ local supersymmetry in spite of the absence of gravitini. The only propagating field is a spin–$\frac{1}{2}$ Dirac spinor χ_A with a possible mass term determined by the AdS_3 cosmological constant. The correspondence with $D=3+1$ SUGRA yields an interpretation of χ_A in terms of the radial component of the $D=3+1$ gravitino. The SUSY parameter ϵ_A in $D=2+1$ is proportional to χ_A.
- Asymptotically AdS_4 solutions featuring the correct boundary geometry in the Fefferman-Graham parametrization comprise the "ultraspinning limit" of the AdS_4-Kerr black hole, or an AdS_3 slicing of AdS_4 (black string) [R. Emparan, G. T. Horowitz, R. C. Myers, JHEP **0001** (2000) 021, hep-th/9912135], with a BTZ black hole [M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992), hep-th/9204099] on the boundary.

Outlook

- Generalization to $\mathcal{N} > 2$ supersymmetry
  With J. Zanelli and R. Noris we are working on a generalization to $\mathcal{N} > 2$ supersymmetry of the model. This allows e.g. the addition of a non-Abelian gauge field, and hence the sudy of the spin-orbit interaction and the quantum spin Hall effect, first postulated in graphene in 2005 [C.L. Kane, E.J. Mele, PRL 95 (22), 226081, arXiv:cond-mat/0411737], but more easily testable in small gap semiconductors like Hg Te/Cd Te (mercury-telluride/ cadmium-telluride) [M. König et al., Science Express Research Articles. 318 (5851): 766770, arXiv:0710.0582 [cond-mat.mes-hall]] with very strong spin-orbit coupling. It also allows to make contact with the ABJM model [O. Aharony, O. Bergman, D.L. Jafferis, J. Maldacena, JHEP, 2008 (10): 091, arXiv:0806.1218 [hep-th]].

- BRST covariant formulation
  With P.A. Grassi we are giving a BRST covariant formulation for the AVZ gauge fixing.

- Properties of topological insulators
  Impurities (doping) act as conical singularities and generate non-trivial holonomies. More in general, with R. Olea and J. Zanelli we are studying the topological properties of the 2+1 dimensional theory, such as domain walls. Indeed, an important feature of graphene is that the constant $\bar{\chi}\chi$, determined in terms of the torsion, corresponds to the difference of pseudospin occupation numbers between sites A and B, and plays the role of a topological index. We are interested in the behaviour of graphene at the boundary of 1+1 dimensional interfaces separating regions where it jumps, breaking the Nieh-Yan-Weyl symmetry [H.T.Nieh, M.L.Yan, Ann.Phys. 138, 237 (1982)].

- Noncommutative generalizations
  Recently it has been observed [A. Iorio, P. Pais, arXiv:1902.00116 [hep-th]] that beyond the low energy approximation of linear spectrum described by the Dirac cone, the emergent effective theory of graphene features a certain generalization of the uncertainty principle, compatible with a non-commutative description.

- This top-down approach to graphene is more predictive than the more common bottom-up one, because it is constrained from the properties of the $D = 3+1$ supergravity theory. $\Longrightarrow$ We are discussing with the condensed matter group at the Politecnico di Torino (F. Dolcini, F. Laviano) to see whether it is possible to check some predictions from supergravity on the physical properties of graphene, such as e.g. conductance, transmission coefficients, shear viscosity.

- Study of the transmission of an electron across a barrier in graphene [M.I. Katsnelson et al., Nature Phys. 2, 620 (2006), cond-mat/0604323; A. Calogeracos, Nature Phys. 2, 579 (2006); A.V. Shytov et al., Phys. Rev. Lett. 101, 156804 (2008), arXiv:0808.0488 [cond-mat.mes-hall]]. For bilayer graphene: on the supergravity side, could it correspond to wormholes between two disjoint 2+1 boundaries of 3+1 space-time?

- Application of this formalism to Weyl semimetals, analogous to graphene in $D = 3 + 1$, because it would be related to a supergravity theory in the bulk of AdS_5 [Y.M.P. Gomes, J. A. Helayel-Neto, Phy. Lett.B:777, 2018, 275-280, arXiv:1711.03220 [hep-th]].