Open String Background Independence

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Perturbative string theory starts with a choice of spacetime background where we embed the relativistic string and quantize the string action.

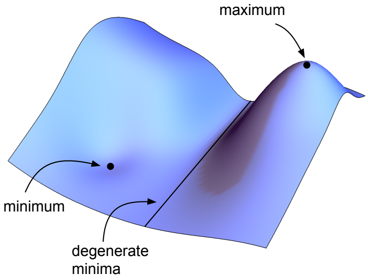
i.e. a choice of worldsheet conformal field theory (CFT)

Background Independence Conjecture:

The choice of CFT merely represents a choice of vacuum state around which we wish to develop string perturbation theory. All CFTs correspond to vacuum states of a single, underlying "nonperturbative string theory."

Pedestrian Interpretation:

$\exists$ a classical potential for string theory:



Worldsheet CFTs correspond to critical points of the potential.

This picture assumes a description of string theory in terms of a field theory action.

$\rightarrow$ String Field Theory

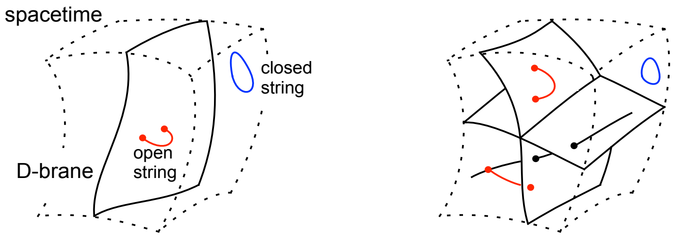
Closed string (or spacetime) background independence is hard, because closed string field theory is complicated.

However, it has been shown that closed string backgrounds which are related by infinitesimal deformation are part of the same theory (Sen, Zwiebach, 90's).

It is not known whether closed string field theory can simultaneously describe all classical closed string backgrounds of a given string theory. It is not even known if it can simultaneously describe any two backgrounds which are a "finite distance" apart.

Open String Background Independence

In a given closed string background, there is still a rich variety of open string vacua. $\rightarrow$ D-branes



Open strings are worldsheets with boundary.

Inside, the worldsheet theory is the same as for the closed string ("bulk CFT"). At the boundary, we have to choose consistent boundary conditions.

This choice defines a boundary conformal field theory (BCFT).

The question is, does open string field theory possess vacua representing all D-brane systems in a given closed string background?

Comment 1:

Tree-level scattering of open strings only produces open strings.

$\therefore$ a classical field theory of open strings is a consistent concept on its own, and we can ask about the vacua of this theory. At the classical level, we do not produce closed string fluctuations or require closed string fields.

Comment 2:

We discuss open bosonic strings, for simplicity.

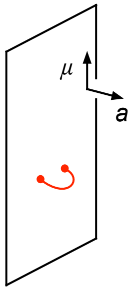
This means: $D = 26$, no fermions, no SUSY.

All D-branes are classically unstable; they contain tachyons in their spectrum of fluctuations.

Open String Field

Open string field theory is the field theory of fluctuations of a "reference" D-brane in a fixed closed string background. The fluctuation fields are given by the open strings which attach to the reference D-brane.

Example: Dp-brane



Dp-brane

$m^2 = -1$:	tachyon	$\rightarrow T(x) =$	tachyonic scalar field
$m^2 = 0$:	$(p+1)$-dim. photon	$\rightarrow A_\mu(x) =$	gauge field $\mu = 0, \ldots, p$
	$25 - p$ massless spin 0 particles	$\rightarrow \phi_a(x) =$	massless scalars $a = p+1, \ldots, 26$
$m^2 > 0$:	massive higher spin particles	$\rightarrow$	higher rank tensor fields

Technical characterization:

An open string field is a state in the vector space $\mathcal{H}$ of the worldsheet BCFT characterizing the reference D-brane.

(idea: 1st $\to$ 2nd quantization; quantum state $\to$ classical field.)

$$\mathrm{BCFT} = \mathrm{BCFT}^{\text{“matter”}} \otimes \mathrm{BCFT}^{\text{ghost}}$$

bc ghosts from
FP gauge fixing

quantizing string
embedding coordinates

The vector space has a $\mathbb{Z}$ grading: ghost number $= \#c's - \#b's$

Dynamical Open String Field:

$$\Psi \in \mathcal{H}, \quad gh\#(\Psi) = 1$$

Example: Dp-brane

"Matter" factor of BCFT is defined by $p+1$ free massless bosons subject to Neumann b.c. at open string boundary

$$X^\mu(z,\bar{z}) \qquad \partial_\perp X^\mu|_{\text{bdry}} = 0 \qquad \mu = 0, ..., p$$

complex coordinates on worldsheet

and $25-p$ free massless bosons subject to Dirichlet b.c.

$$X^a(z,\bar{z}) \qquad \partial_\parallel X^a|_{\text{bdry}} = 0 \qquad \mu = p+1, ..., 25$$

Ghost factor of BCFT is defined by anticommuting ghost fields on worldsheet:

$$b(z), c(z)$$

The ghost factor is the same for all D-brane systems.

The worldsheet fields can be expanded into Fourier modes, and we may find a basis for $\mathcal{H}$ by acting mode oscillators on the vacuum.

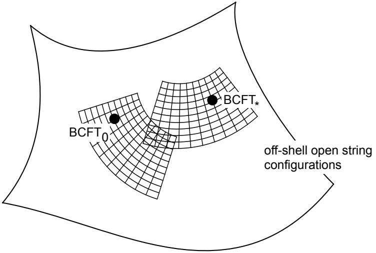
In this way, the dynamical string field can be written

$$\Psi = \int \frac{d^{p+1}k}{2\pi}\Big(\underbrace{T(k)}_{\text{tachyon}} + \underbrace{A_\mu(k)}_{\text{gauge field}}\alpha^\mu_{-1} + \underbrace{\phi_a(k)}_{\text{massless scalars}}\alpha^a_{-1} + \underbrace{\beta(k)}_{\text{auxiliary field}} c_0 b_{-1} + \ldots\Big)e^{ik\cdot x}c_1|0\rangle$$

Comment: The nature of the open string field depends on the choice of reference D-brane.

Is there a reference D-brane which defines the "fundamental" field variable for the purposes of characterizing the space of open string backgrounds? No.

A priori, the true configuration space of open string theory will have to be covered by a patchwork of coordinate systems corresponding to the fluctuation fields of each possible D-brane configutation.



Open Bosonic String Field Theory

Let us settle on some reference D-brane BCFT_0, and $\Psi \in \mathcal{H}_0$ is the fluctuation field of this D-brane.

Linearized EOM and gauge symmetry:

$$\underbrace{Q}_{\substack{\text{BRST}\\ \text{operator, } gh\#1}}\Psi = 0, \qquad \delta\Psi = Q\underbrace{\Lambda}_{\substack{\text{gauge}\\ \text{parameter, } gh\#0}}$$

Nonlinear EOM and gauge symmetry:

$$Q\Psi + \underbrace{\Psi^2}_{\substack{\text{associative}\\ \text{“star'' product}}} = 0, \qquad \delta\Psi = Q\Lambda + [\Psi, \Lambda]$$

Chern-Simons-like action:

$$S = \underbrace{\mathrm{Tr}}_{\text{trace}}\left[\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right]$$

The vacua of the BCFT_0 open string field theory are characterized by solutions to the equations of motion:

$$Q\Psi + \Psi^2 = 0$$

Question: What portion of the set of all D-brane systems in a common closed string background are included as solutions to these equations of motion?

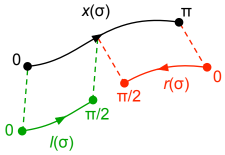
Some history on this question

Witten formulates open bosonic string field theory in 1986.

- First 15 years: Who knows? By the way, what is a D-brane?
- Next 20 years: The fluctuation fields of the reference D-brane describe some nontrivial vacua, in particular other D-brane configurations with lower energy. Established by:
  1. Numerical Solution, component field by component field in the Fock space basis. Sen, Zwiebach and others early 2000s.
  2. Analytic Solution. Schnabl, 2005.
- Now: We will argue that *all* open string vacua are realized as classical solutions around *any* reference D-brane. T.E. with Maccaferri.

More on the star product and trace

Schrodinger representation



String field can be viewed as a functional of a curve in spacetime:

$$\Psi[x(\sigma)] = \langle x(\sigma)|\Psi\rangle \qquad x(\sigma) = X(z,\bar{z})|_{z=e^{i\sigma}}$$

(ignoring Lorentz indices and ghosts.) Split curve into left and right halves:

$$\begin{aligned} l(\sigma) &= x(\sigma), \\ r(\sigma) &= x(\pi-\sigma), \end{aligned} \qquad \sigma \in [0,\pi/2]$$

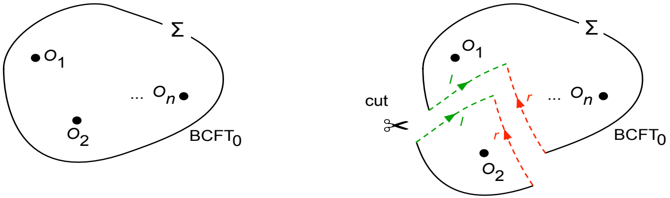
and reexpress the functional $\Psi[x(\sigma)]$ as $\Psi[l(\sigma), r(\sigma)] =$ "matrix".

Star product: $AB[l,r] = \int [dw] A[l,w] B[w,r]$

Trace: $\mathrm{Tr}[A] = \int [dw] A[w,w]$

Schrodinger functional of a surface

1) Consider a BCFT_0 correlation function on a surface Σ, $\langle O_1 O_2 \dots O_n \rangle_{\Sigma}^{\mathrm{BCFT}_0}$:

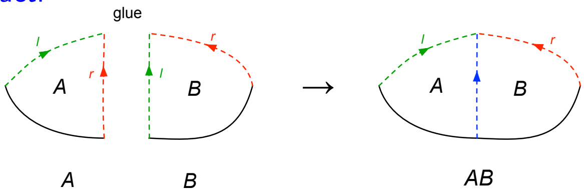


2) Cut out a piece of Σ along two parameterized curves l, r which meet at a common point in the interior.

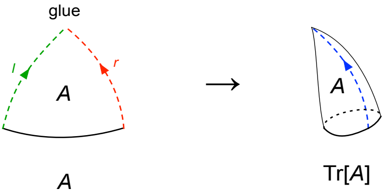
3) Carry out the worldsheet path integral on the excised portion of Σ, with BCFT_0 b.c. on the open string boundary and b.c. $l(\sigma)$, $r(\sigma)$ for the worldsheet fields on the curves l, r.

4) This defines a functional of $l(\sigma)$, $r(\sigma)$ which characterizes an open string field Ψ.

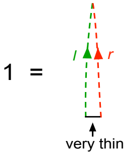
Star product:



Trace:

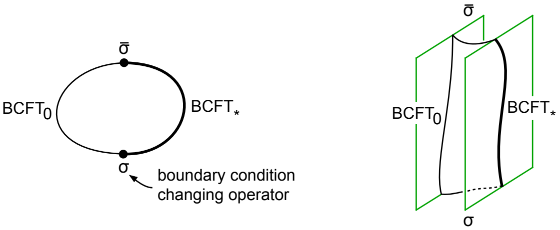


Identity:



Stretched strings and boundary condition changing operators

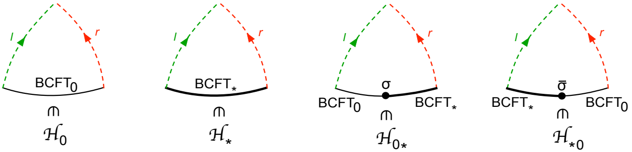
It is possible to consider worldsheets where part of the open string boundary carries BCFT_0 b.c., and another part $\mathrm{BCFT}_\star$ b.c.



Some extra data is needed to specify what happens when these boundary conditions meet $\rightarrow$ boundary condition changing operators

Boundary condition changing operators specify the state of a streched string connecting two D-branes.

Cutting the stretched string surface in different ways gives string fields in four vector spaces:



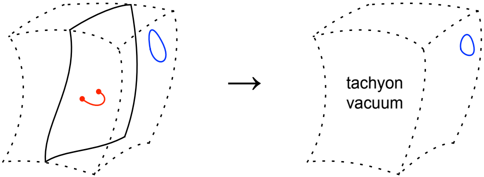
Star products between these sectors can be defined if the boundary conditions match when gluing the surfaces together:

$$
\begin{aligned}
\mathcal{H}_0 * \mathcal{H}_0 &\in \mathcal{H}_0, \qquad & \mathcal{H}_\star * \mathcal{H}_\star &\in \mathcal{H}_\star \\
\mathcal{H}_0 * \mathcal{H}_{0\star} &\in \mathcal{H}_{0\star}, \qquad & \mathcal{H}_{0\star} * \mathcal{H}_\star &\in \mathcal{H}_{0\star} \\
\mathcal{H}_{\star 0} * \mathcal{H}_0 &\in \mathcal{H}_{\star 0}, \qquad & \mathcal{H}_\star * \mathcal{H}_{\star 0} &\in \mathcal{H}_{\star 0} \\
\mathcal{H}_{\star 0} * \mathcal{H}_{0\star} &\in \mathcal{H}_\star, \qquad & \mathcal{H}_{0\star} * \mathcal{H}_{\star 0} &\in \mathcal{H}_0
\end{aligned}
$$

Solution of $Q\Psi + \Psi^2 = 0$

Given string field theory formulated around a reference D-brane BCFT_0, we seek a solution $\Psi_\star$ describing a target D-brane $\mathrm{BCFT}_\star$.

First: Every D-brane system in bosonic string theory is classically unstable. The endpoint of this instability is the vacuum where the D-brane has completely annihilated, and we have empty space without open strings. (Sen, 2000).



String field theory formulated on any reference D-brane has a corresponding classical solution Ψ_{tv}:

$$Q\Psi_{\mathrm{tv}} + \Psi_{\mathrm{tv}}^2 = 0$$

called the tachyon vacuum.

We can shift the string field so as to expand around the tachyon vacuum:

$$\Psi = \Psi_{\mathsf{tv}} + \psi$$

The equations of motion become

$$Q_{\Psi_{\mathsf{tv}}}\psi + \psi^2 = 0, \qquad (Q_{\Psi_{\mathsf{tv}}} = Q + [\Psi_{\mathsf{tv}}, \cdot])$$

The shifted equations of motion have a trivial solution

$$\psi = -\Psi_{\mathsf{tv}}$$

describing the BCFT_0 D-brane from the point of view of the tachyon vacuum.

This suggests that we can create the $\mathrm{BCFT}_\star$ D-brane by subtracting the tachyon vacuum of the BCFT_* string field theory.

In terms of the original string field Ψ of the reference D-brane, the solution would be

$$\Psi_\star \overset{?}{=} [\Psi_{\mathrm{tv}}]_{\mathrm{BCFT}_0} - [\Psi_{\mathrm{tv}}]_{\mathrm{BCFT}_\star}$$

We first destroy the reference D-brane by condensing to the tachyon vacuum, and then build the target D-brane from there.

Problem: The two tachyon vacuum solutions live in different vector spaces, and cannot be subtracted.

Resolution: Introduce $\Sigma \in \mathcal{H}_{0\star}$ and $\overline{\Sigma} \in \mathcal{H}_{\star 0}$ so that

$$\Psi_\star = \Psi_{\mathrm{tv}} - \Sigma \Psi_{\mathrm{tv}} \overline{\Sigma}$$

is consistently defined as a state in the reference boundary conformal field theory.

The equations of motion require that

$$Q_{\Psi_{\mathrm{tv}}} \Sigma = Q_{\Psi_{\mathrm{tv}}} \overline{\Sigma} = 0$$
$$\overline{\Sigma}\Sigma = 1$$

Interpretation: Σ and $\overline{\Sigma}$ are "intertwiners" which allow us express the variables of the target D-brane in terms of the variables of the reference D-brane.

i.e. stretched strings provide a dictionary between the degrees of freedom of different D-brane systems.

This means that a D-brane, in its fluctuations, contains all information about every other D-brane system which shares the same closed string background

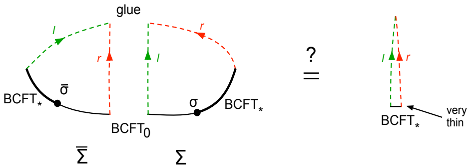
In fact, we can transform between the "coordinate systems" defined by the fluctuation fields of different D-brane systems using the field redefinition

$$[\Psi]_{\mathrm{BCFT}_0} = \Psi_\star + \Sigma[\Psi]_{\mathrm{BCFT}_\star}\overline{\Sigma}$$

With the fields related in this way, one can show that the Chern-Simons-like actions of the two D-brane systems are equal up to an additive constant:

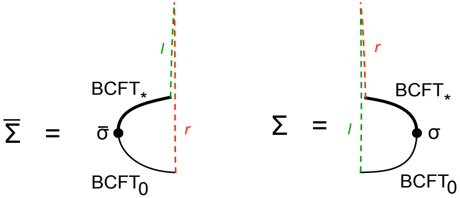
$$S([\Psi]_{\mathrm{BCFT}_0}) = S([\Psi]_{\mathrm{BCFT}_\star}) + \mathit{constant}$$

Problem: How do we concretely realize the identity $\overline{\Sigma}\Sigma = 1$?

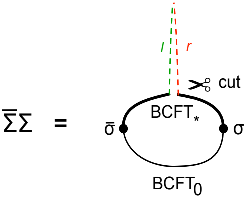


Seems that $\overline{\Sigma}$ and Σ cannot contain any surface, but then there is no room for a change of boundary condition.

Resolution: Take $\bar{\Sigma}$ and Σ to have the following curious shape:



Star product produces an “appendage” which can be cut away from the surface:



With this definition Σ multiplied by $\overline{\Sigma}$ (in the opposite order) does not equal the identity:

$$\Sigma\overline{\Sigma} \neq 1$$

Therefore Σ and $\overline{\Sigma}$ multiply as non-unitary isometries.

This implies that the fields of the target D-brane are mapped 1-to-1 into a subset of fields of the reference D-brane. Switching the roles of BCFT_0 and $\mathrm{BCFT}_\star$, the fields of the reference D-brane can also be mapped 1-to-1 into a subset of fields of the target D-brane.

Therefore, the fields of the reference D-brane can be mapped into a subset of themselves without losing any physical information. In a sense, the string field covers all open string vacua with room to spare.

Thank you!