

Quantum space-time, higher spin and gravity from the IKKT matrix model

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COST Action MP 1405
Quantum Structure of Spacetime

how to formulate **quantum** theory of **spacetime** & **gravity**?

guidelines:

- simple
- gauge theory (Minkowski signature!)
- finite dof per volume (Planck scale!)
→ underlying d.o.f. **non-geometric**
- GR established only in IR regime
space-time & gravity may **emerge** from other d.o.f.

(cf. strong interactions from QCD !)

Matrix Models (of Yang-Mills type)

$S = \text{Tr}([X^\mu, X^\nu][X_\mu, X_\nu] + \dots)$ provide such models!

- simple
- describe dynamical NC / fuzzy spaces, **gauge theory**

$$X^a \rightarrow U^{-1} X^a U$$

- well suited for quantization: $\int dX e^{-S[X]}$
 - generic models: serious UV/IR mixing problem
 - preferred model: maximal SUSY = **IKKT model**
shares features of string theory, cut the “landscape”
- **gravity?**

summary of recent results to be discussed:

- (3+1)-dim. covariant quantum space-time solution
(FLRW cosmology, Big Bounce)
- tower of higher-spin modes, truncated at n .
→ higher-spin gauge theory, all d.o.f. required for gravity
- propagation governed by universal **dynamical metric**
(Lorentz invar. only partially manifest)
- metric perturbations → massless graviton & scalar
- analog of the linearized Einstein-Hilbert action
(expected to be induced upon quantization)

HS, arXiv:1606.00769

HS, arXiv:1710.11495

M. Sperling, HS arXiv:1806.05907

M. Sperling, HS arXiv:1901.03522

outline:

- the IKKT matrix model & NC gauge theory
- *4D covariant quantum spaces*:
fuzzy H_n^4 , cosmological space-time $\mathcal{M}_n^{3,1}$
- fluctuations \rightarrow higher spin gauge theory
- metric perturbations
linearized Einstein-Hilbert action & gravity

The IKKT model

IKKT or IIB model

Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[Y, \Psi] = -\text{Tr} \left([Y^a, Y^b][Y^{a'}, Y^{b'}] \eta_{aa'} \eta_{bb'} + m^2 Y^a Y_a + \bar{\Psi} \gamma_a [Y^a, \Psi] \right)$$

$$Y^a = Y^{a\dagger} \in \text{Mat}(N, \mathbb{C}), \quad a = 0, \dots, 9, \quad N \rightarrow \infty$$

gauge invariance $Y^a \rightarrow U Y^a U^{-1}$, $SO(9, 1)$, SUSY

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:
 - $Y^a + m^2 Y^a = 0$, □ $\equiv \eta_{ab} [Y^a, [Y^b, \cdot]]$
- quantization: $Z = \int dY d\Psi e^{iS[Y]}$, SUSY essential

strategy:

- look for solutions \rightarrow space(time)
- fluctuations \rightarrow gauge theory, dynamical geometry, **gravity ?!**
- matrix integral = (Feynman) path integral, **incl. geometry**

class of solutions: fuzzy spaces = **quantized symplectic manifolds**

$$X^a \sim x^a: \quad \mathcal{M} \hookrightarrow \mathbb{R}^{9,1}$$

$$[X^a, X^b] \sim i\theta^{ab}(x) \quad \dots \text{ (quantized) Poisson tensor}$$

algebra of functions on fuzzy space: $\text{End}(\mathcal{H})$

- Moyal-Weyl quantum plane \mathbb{R}_θ^4 :

$$[X^a, X^b] = i\theta^{ab} \mathbf{1}$$

quantized symplectic space (\mathbb{R}^4, ω)

admits translations $X^a \rightarrow X^a + \mathbf{c}^a \mathbf{1}$, **no rotation invariance**

- fuzzy 2-sphere S_N^2

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \quad [X_i, X_j] = i\epsilon_{ijk} X_k$$

fully **covariant** under $SO(3)$

(Hoppe; Madore)

fluctuations in M.M. on fuzzy space \rightarrow NC gauge theory

derivations:

$$[X^\mu, \phi] =: i\theta^{\mu\nu} \partial_\nu \phi$$

consider **fluctuations** around background X^a

$$Y^a = X^a + \mathcal{A}^a$$

$$[Y^\mu, \phi] = i\theta^{\mu\nu} D_\nu \phi, \quad D_\mu = \partial_\mu + i[A_\mu, \cdot]$$

$$[Y^\mu, Y^\nu] = i\theta^{\mu\nu} + i\theta^{\mu\mu'} \theta^{\nu\nu'} \underbrace{(\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}])}_{F_{\mu'\nu'}}$$

$$S = \text{Tr}([Y^\mu, Y^\nu][Y_\mu, Y_\nu]) \sim \int F_{\mu\nu} F^{\mu\nu} + c.$$

\rightarrow YM gauge theory

$$A_\mu \rightarrow U^{-1} A_\mu U + U^{-1} \partial_\mu U$$

\leftrightarrow dynamical geometry ("emergent gravity")

emergent gravity on deformed \mathcal{M}_θ^4 ?

H.S. arXiv:1003.4134 ff

cf. H.Yang, hep-th/0611174 ff

eff. **metric** encoded in $\square = [X_a, [X^a, \cdot]] \sim -e^\sigma(x)\Delta_G$

$$G^{\mu\nu} = e^{-\sigma} \theta^{\mu\mu'} \theta^{\mu\mu'} g_{\mu'\nu'}$$

fluctuations $X^a + \mathcal{A}^a(X) \rightarrow$ dynamical metric

cf. Rivelles hep-th/0212262

bare M.M action: 2 Ricci-flat metric fluctuations, not full Einstein eq.

quantum effects \rightarrow **induced gravity** (Sakharov)

problems:

- $\theta^{\mu\nu}$ breaks Lorentz invariance \rightarrow other terms possible
(e.g. $R_{\mu\nu\alpha\beta} \theta^{\mu\nu} \theta^{\alpha\beta}$ D. Klammer, H.S. arXiv:0909.5298)
- huge cosm. constant

issues seem resolved for **covariant quantum spaces**:

4D covariant quantum spaces

- issue: symplectic form ω breaks local (Lorentz/Euclid.) invar.
- avoided on **covariant quantum spaces**

example: fuzzy four-sphere S_N^4

Grosse-Klimcik-Presnajder; Castellino-Lee-Taylor; Medina-o'Connor;
Ramgoolam; Kimura; Karabail-Nair; Zhang-Hu 2001 (QHE) ...

→ higher-spin gauge theory in IKKT HS arXiv:1606.00769

- noncompact H_n^4 → higher-spin gauge theory

M. Sperling, HS arXiv:1806.05907

- projection of H_n^4 → cosmological space-time $\mathcal{M}_n^{3,1}$

HS, arXiv:1710.11495, arXiv:1709.10480

- spin 2 modes on $\mathcal{M}_n^{3,1}$ → gravity

M. Sperling, HS arXiv:1901.03522

Euclidean fuzzy hyperboloid H_n^4 ($=EAdS_n^4$)

Hasebe arXiv:1207.1968

\mathcal{M}^{ab} ... hermitian generators of $\mathfrak{so}(4, 2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) .$$

choose “short” discrete unitary irreps \mathcal{H}_n $\eta^{ab} = \text{diag}(-1, 1, 1, 1, 1, -1)$ (“minireps”, doubletons)

special properties:

- irreps under $\mathfrak{so}(4, 1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\text{spec}(\mathcal{M}^{05}) = \{E_0, E_0 + 1, \dots\}, \quad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n + 1$ -dim. irrep of $SU(2)_L$: fuzzy S_n^2

fuzzy hyperboloid H_n^4

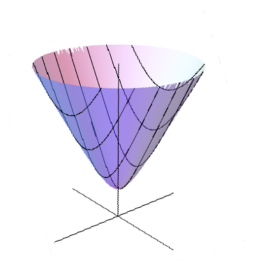
def.

$$\begin{aligned} X^a &:= r\mathcal{M}^{a5}, & a = 0, \dots, 4 \\ [X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab} \end{aligned}$$

5 hermitian generators X^a satisfy

(cf. Snyder)

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \quad R^2 = r^2(n^2 - 4)$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4, 1)$

note: induced metric = Euclidean AdS^4

oscillator construction: 4 bosonic oscillators $[\psi_\alpha, \bar{\psi}^\beta] = \delta_\alpha^\beta$

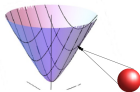
$\mathcal{H}_n =$ suitable irrep in Fock space

Then

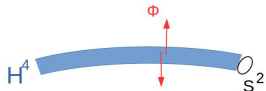
$$\mathcal{M}_{ab} = \bar{\psi} \Sigma_{ab} \psi, \quad \gamma_0 = \text{diag}(1, 1, -1, -1)$$

$$X^a = r \bar{\psi} \gamma^a \psi$$

fact: $H_n^4 =$ quantized $\mathbb{C}P^{1,2} = S^2$ bundle over H^4 , selfdual $\theta^{\mu\nu}$



functions on $H_n^4 \stackrel{loc}{\cong} S^2 \times H^4 =$ harmonics on $S^2 \times$ functions on H^4



local stabilizer acts on $S^2 \rightarrow$ harmonics = higher spin modes

fuzzy "functions" on H_n^4 :

$$(End(\mathcal{H}_n) \rightsquigarrow) \quad HS(\mathcal{H}_n) = \int_{\mathbb{C}P^{1,2}} f(m) |m\rangle \langle m| \cong \bigoplus_{s=0}^n \mathcal{C}^s$$

\mathcal{C}^0 = scalar functions on H^4 : $\phi(X)$

\mathcal{C}^1 = selfdual 2-forms on H^4 : $\phi_{\mu\nu}(X)\theta^{\mu\nu} = \begin{array}{|c|} \hline \square \\ \hline \end{array}$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \bigoplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \ni \theta^{\mu_1\nu_1} \dots \theta^{\mu_s\nu_s}$

higher spin modes = would-be KK modes on S^2

i.e. higher spin theory, truncated at n

M. Sperling, HS arXiv:1806.05907

H_n^4 is starting point for cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell higher-spin fluctuations obtained
- spin 2 metric fluctuations \rightarrow gravity (linearized)

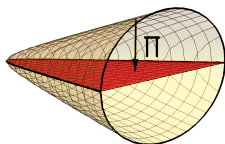
open FRW universe from H_n^4

HS arXiv:1710.11495

$\mathcal{M}_n^{3,1} = H_n^4$ projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \mathbb{C}P^{1,2} \rightarrow H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} .$$

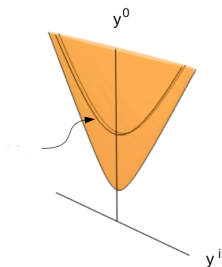
induced metric has Minkowski signature!



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$Y^\mu := X^\mu, \quad \text{for } \mu = 0, 1, 2, 3 \quad (\text{drop } X^4)$$

geometric properties:



- $SO(3, 1)$ manifest \Rightarrow foliation into $SO(3, 1)$ -invariant space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$d\Sigma^2$... $SO(3, 1)$ -invariant metric on space-like H^3

functions on $\mathcal{M}^{3,1}$:

generated by $X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ and $T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$, with CR

$$\begin{aligned} \{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu} \\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu} \\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2 R^2}\theta^{\mu\nu} \end{aligned}$$

constraints

$$\begin{aligned} x_\mu x^\mu &= -R^2 \cosh^2(\eta), & x^4 &= R \sinh(\eta) \\ t_\mu t^\mu &= r^{-2} \cosh^2(\eta), \\ t_\mu x^\mu &= 0, \\ t_\mu \theta^{\mu\alpha} &= -\sinh(\eta)x^\alpha, \\ x_\mu \theta^{\mu\alpha} &= -r^2 R^2 \sinh(\eta)t^\alpha, \\ \eta_{\mu\nu} \theta^{\mu\alpha} \theta^{\nu\beta} &= R^2 r^2 \eta^{\alpha\beta} - R^2 r^4 t^\alpha t^\beta - r^2 x^\alpha x^\beta \\ \theta^{\mu\nu} \theta_{\mu\nu} &= 2R^2 r^2 (2 - \cosh^2(\eta)) \end{aligned}$$

hence: t^μ ... space-like S^2

self-duality on $H^4 \Rightarrow \theta^{\mu\nu} = c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta} x_\alpha t_\beta$

functions as higher-spin modes:

$$\phi = \sum_{s=0}^n \phi^{(s)} \in \text{End}(\mathcal{H}_n), \quad \phi^{(s)} \in \mathcal{C}^s$$

2 points of view:

- functions on H_n^4 : full $SO(4, 1)$ covariance
represent $\phi^{(s)}$ as

$$\begin{aligned} \phi_{a_1 \dots a_s}^H &\propto \{x^{a_1}, \dots \{x^{a_s}, \phi^{(s)}\} \dots\}_0 \\ \phi^{(s)} &= \{x^{a_1}, \dots \{x^{a_s}, \phi_{a_1 \dots a_s}^H\} \dots\} \end{aligned}$$

- functions on $\mathcal{M}_n^{3,1}$: reduced $SO(3, 1)$ covariance

$$\phi^{(s)} = \phi_{\mu_1 \dots \mu_s}^{(s)}(x) t^{\mu_1} \dots t^{\mu_s}$$

$t_\mu x^\mu = 0 \Rightarrow$ "space-like gauge"

$$x^{\mu_i} \phi_{\mu_1 \dots \mu_s}^{(s)} = 0$$

(\rightarrow no ghosts!)

$\mathcal{M}^{3,1}$ realization in IKKT model:

background solution:

$$T^\mu := \frac{1}{R} \mathcal{M}^{\mu 4}$$

satisfies

$$\square T^\mu = 3R^{-2} T^\mu, \quad \square = [T^\mu, [T_\mu, \cdot]]$$

- $[\square, S^2] = 0, \quad S^2 = [\mathcal{M}^{ab}, [\mathcal{M}_{ab}, \cdot]] + r^{-2} [X_a, [X^a, \cdot]]$

... **spin Casimir**, selects spin s sectors \mathcal{C}^s

\Rightarrow higher-spin expansion $\phi = \phi(X) + \phi_\mu(X) T^\mu + \dots$ on $\mathcal{M}^{3,1}$

- $\square \sim \alpha^{-1} \square_G$ encodes eff. FRW metric $ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$, asymptotically coasting $a(t) \propto t$
- Big Bounce, initial $a(t) \sim t^{1/5}$ singularity

fluctuations & higher spin gauge theory

$$S[Y] = \text{Tr}(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S_N^4, H_n^4, \mathcal{M}_n^{3,1}$

add **fluctuations** $Y^a = \bar{Y}^a + \mathcal{A}^a,$

gauge trafos $\mathcal{A}^a \rightarrow [\Lambda, \mathcal{A}^a] + [\Lambda, \bar{Y}^a], \quad \Lambda \in \text{End}(\mathcal{H})$

expand action to second order in \mathcal{A}^a

$$S[Y] = S[\bar{Y}] + \frac{2}{g^2} \text{Tr} \mathcal{A}_a \left(\underbrace{\left(\square + \frac{1}{2} m^2 \right) \delta_b^a + 2[[\bar{Y}^a, \bar{Y}^b], \cdot]}_{\mathcal{D}^2} - \underbrace{[\bar{Y}^a, [\bar{Y}^b, \cdot]]}_{\text{g.f.}} \right) \mathcal{A}_b$$

\mathcal{A}_a ... \mathfrak{hs} -valued field on \mathcal{M} , incl. spin 2

diagonalization & eigenmodes on $\mathcal{M}_n^{3,1}$: background $\bar{Y}^\mu = T^\mu$

M. Sperling, HS: arXiv:1901.03522

symmetry: only space-like $SO(3, 1)$

underlying $SO(4, 1)$ & $SO(4, 2)$ extremely useful

- ansatz:

$$\begin{aligned}\mathcal{A}_\mu^{(+)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_+ \\ \mathcal{A}_\mu^{(-)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_- .\end{aligned}$$

can show using $\mathfrak{so}(4, 2)$ structure: are **eigenmodes**

$$\begin{aligned}\mathcal{D}^2 \mathcal{A}_\mu^{(+)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(+)} \left[\left(\square + \frac{2s+5}{R^2} \right) \phi^{(s)} \right] \\ \mathcal{D}^2 \mathcal{A}_\mu^{(-)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(-)} \left[\left(\square + \frac{-2s+3}{R^2} \right) \phi^{(s)} \right] .\end{aligned}$$

→ 2 physical modes in

$$\mathcal{H}_{\text{phys}} = \{ \mathcal{D}^2 \mathcal{A} = 0, \mathcal{A} \text{ gauge fixed} \} / \{ \text{pure gauge} \}$$

- pure gauge mode

$$\mathcal{A}_\mu^{(g)}[\phi^{(s)}] = \{t_\mu, \phi^{(s)}\}$$

- time-like mode $\mathcal{A}_\mu^{(\tau)}[\phi^{(s)}] := x_\mu \phi^{(s)}$
 off-shell, time-like d.o.f. (not in $\mathcal{H}_{\text{phys}}$ (no proof))
conjecture: **no ghosts** (cf. YM !)
 - all physical on-shell modes found,
 2 physical propagating modes for each spin $s \leq n$
 - same speed (of light) for all physical modes
 even though $SO(3, 1)$ only space-like
- Lorentz-violating structures: $x_\mu \{t^\mu, \cdot\} \sim \frac{\partial}{\partial \tau}$... time-like VF
 (cosmic background!)

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(x)$ (e.g. transversal fluctuation)

kinetic term $-Tr[T^\alpha, \phi][T_\alpha, \phi] \sim \int e^\alpha \phi e_\alpha \phi = \int \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

vielbein

$$\begin{aligned} e^\alpha &:= \{T^\alpha, \cdot\} = e^{\alpha\mu} \partial_\mu \\ e^{\alpha\mu} &= \sinh(\eta) \eta^{\alpha\mu} \end{aligned}$$

metric

$$\gamma^{\mu\nu} = \eta_{\alpha\beta} e^{\alpha\mu} e^{\beta\nu}$$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$e^\alpha = \{T^\alpha + \mathcal{A}^\alpha, \cdot\} = e^{\alpha\mu} [\mathcal{A}] \partial_\mu$$

$$\delta_{\mathcal{A}} \gamma^{\mu\nu} \sim \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$$

linearize & average over fiber $\rightarrow h^{\mu\nu} = [\delta_{\mathcal{A}} \gamma^{\mu\nu}]_0$

coupling to matter:

$$S[\text{matter}] \sim \int_{\mathcal{M}} d^4x h^{\mu\nu} T_{\mu\nu}$$

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higher-spin gravity on $\mathcal{M}^{3,1}$

perturbed metric: $h^{\mu\nu} \propto \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$

contains all dof required for gravity

5+1 off-shell dof from $\mathcal{A}^{(-)}[\phi^{(2)}]$ and $\mathcal{A}^{(+)}[\phi^{(0)}]$, 3 pure gauge (!)

Ricci tensor:

$$\mathcal{R}_{(\text{lin})}^{\mu\nu}[h[\mathcal{A}]] \approx \frac{1}{2} \underbrace{\square h_{\mu\nu}[\mathcal{A}]}_{h_{\mu\nu}[\mathcal{D}^2 \mathcal{A}]} - \frac{1}{4} (\{t_\mu, \{t_\nu, h\}\} + (\mu \leftrightarrow \nu))$$

(up to cosm. scales)

on-shell (vacuum) in M.M.:

$\mathcal{A}^-[\phi^{(2,0)}]$: $h \approx 0 \Rightarrow \mathcal{R}_{(\text{lin})}^{\mu\nu} \approx 0$... 2 graviton modes (**massless** !)

$\mathcal{A}^-[\phi^{(2,1)}]$: $h \approx 0 \Rightarrow \mathcal{R}_{(\text{lin})}^{\mu\nu} \approx 0$... trivial (on-shell)

extra scalar mode(s): $\mathcal{A}^-[\phi^{(2,2)}], \mathcal{A}^+[\phi^{(0)}]$

(approx. identical $\mathcal{R}_{(\text{lin})}^{\mu\nu} \neq 0$)

gauge transformations

act on functions $\phi \in \mathcal{C}$ as $\phi \mapsto \{\Lambda, \phi\}$

spin 1 transformations: $\Lambda = v^\mu(x)t_\mu \in \mathcal{C}^1$:

$$\begin{aligned} \{v^\mu t_\mu, \phi\}_0 &= [t^\mu \theta^{\alpha\beta}]_0 \partial_\alpha v_\mu \partial_\beta \phi + v^\mu \{t_\mu, \phi\}_0 \\ &= \frac{1}{3} (\sinh(\eta) (3v^\mu \partial_\mu + (\operatorname{div} v)\tau - (\tau v^\mu) \partial_\mu) \phi + x_\gamma \varepsilon^{\gamma\mu\alpha\beta} \partial_\alpha v_\mu \partial_\beta \phi) \end{aligned}$$

includes 3 (rather than 4) diffeomorphisms !

due to invar. symplectic volume on $\mathbb{C}P^{1,2}$

act on gauge fields \mathcal{A}^μ as $\mathcal{A}^\mu \mapsto \{\Lambda, T^\mu + \mathcal{A}^\mu\}$

pure gauge modes: $\mathcal{A}_\mu^{(g)} = \{t_\mu, \Lambda\}$,

$$\text{e.g. } h_{\mu\nu}^{(g)} \approx \partial_\mu \mathcal{A}_\nu + \partial_\nu \mathcal{A}_\mu, \quad \mathcal{A}_\mu = \{x_\mu, \Lambda\}_-$$

linearized Einstein-Hilbert action

- determined by gauge invariance $\delta_\phi \mathbf{S}_{\text{EH}} = 0$

result:

$$\mathbf{S}_{\text{EH}} := \mathbf{S}_1 - \frac{1}{24} \mathbf{S}_h - \frac{1}{R^2} \mathbf{S}_3 - \frac{1}{2R^2} \mathbf{S}_4 + \frac{3}{R^2} \left(\frac{1}{2} \mathbf{S}_{M3} - 2\mathbf{S}_{gf2} + \frac{3}{2} \mathbf{S}_{gf3} \right)$$

\approx linearized Einstein-Hilbert for traceless modes

where

$$\begin{aligned} \mathbf{S}_1 &= \int (h_{\mu\nu} - \frac{1}{3} h \eta_{\mu\nu}) \mathcal{G}_{(\text{lin})}^{\mu\nu} [h_{\alpha\beta} - \frac{1}{3} h \eta_{\alpha\beta}] \\ \mathbf{S}_h &= \int h \square h, \\ \mathbf{S}_3 &= \int \delta g_{\mu\nu}^0 \delta g_0^{\mu\nu}, \\ \mathbf{S}_4 &= \int \delta g_{\mu\nu}^0 \{ \mathcal{M}^{\mu\rho}, \delta g_0^{\rho\nu} \}, \\ \mathbf{S}_{M3} &= \int f_{\mu\nu} \{ \mathcal{M}^{\nu\rho}, h^{\rho\mu} \}, \\ \mathbf{S}_{gf2} &= \int \{ x_\mu, A^\mu \} - D^- \{ t_\rho, A^\rho \}, \\ \mathbf{S}_{gf3} &= \int (D^- \{ t_\nu, A^\nu \}) \square^{-1} (D^- \{ t_\nu, A^\nu \}), \end{aligned}$$

- expected to be induced by quantum effects (cf. [Sakharov 1967](#))

- Ricci-flat vacuum solutions for both bare M.M. and induced S_{EH}
 S_{EH} needed to recover inhomogeneous Einstein eq. $G_{\mu\nu} \propto T_{\mu\nu}$
 \Rightarrow expect \approx linearized GR at intermediate scale,
 good agreement with solar system tests
- model is fully non-linear (to be understood)
- no cosm. const. $\int d^4x \sqrt{g}$
 replaced by YM-action, stabilizes $\mathcal{M}^{3,1}$
 \rightarrow no cosm. const. problem ?!
- significant differences at cosmic scales,
 reasonable cosmology without any fine-tuning !!

bare YM action and gravitons:

can rewrite

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto - \int h^{\mu\nu}[\phi^{(2,0)}] \left(\square - \frac{2}{R^2}\right) (\square_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leading to

$$\left(\square - \frac{2}{R^2}\right) h_{\mu\nu} \sim -(\square_H - 2r^2) T_{\mu\nu}$$

summary

- **matrix models:**
natural framework for quantum theory of space-time & matter
- **4D covariant quantum spaces** \rightarrow higher spin theories
- \exists nice cosmological FRW space-time solutions
reg. BB, finite density of microstates
- Yang-Mills structure \rightarrow **emergent gravity** rather than GR
- good UV behavior (SUSY), well suited for quantization

... seems to work! needs to be elaborated

breaking $SO(4, 1) \rightarrow SO(3, 1)$ and sub-structure

consider

$$D\phi := -i[X^4, \phi], \quad \text{respects } SO(3, 1)$$

acts on spin s modes as follows

$$D = \underbrace{\text{div}^{(3)}\phi}_{D^-\phi} + \underbrace{t^\mu \nabla_\mu^{(3)}\phi}_{D^+\phi} : \mathcal{C}^s \rightarrow \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1}$$

decomposition into $SO(3, 1)$ irreps on $H^3 \subset H^4$

$$\mathcal{C}^{(s)} = \mathcal{C}^{(s,0)} \oplus \mathcal{C}^{(s,1)} \oplus \dots \oplus \mathcal{C}^{(s,s)}$$

D^- resp. D^+ act as

$$D^- : \mathcal{C}^{(s,k)} \rightarrow \mathcal{C}^{(s-1,k-1)}, \quad D^+ : \mathcal{C}^{(s,k)} \rightarrow \mathcal{C}^{(s+1,k+1)} .$$