Phase diagram of modified scalar field theory on fuzzy sphere

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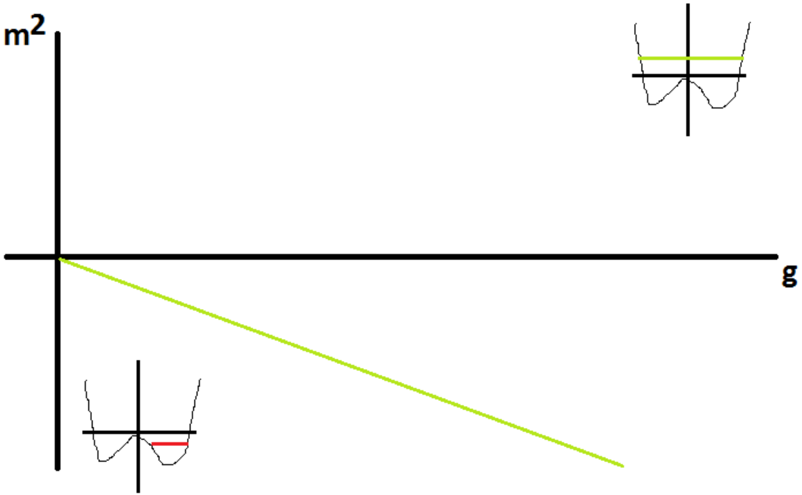
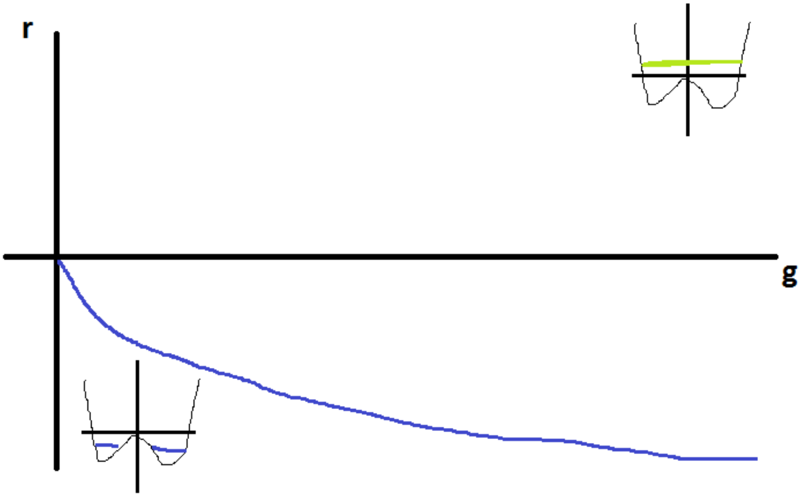
[1711.02008 [hep-th]],[1802.05188 [hep-th]], work in progress

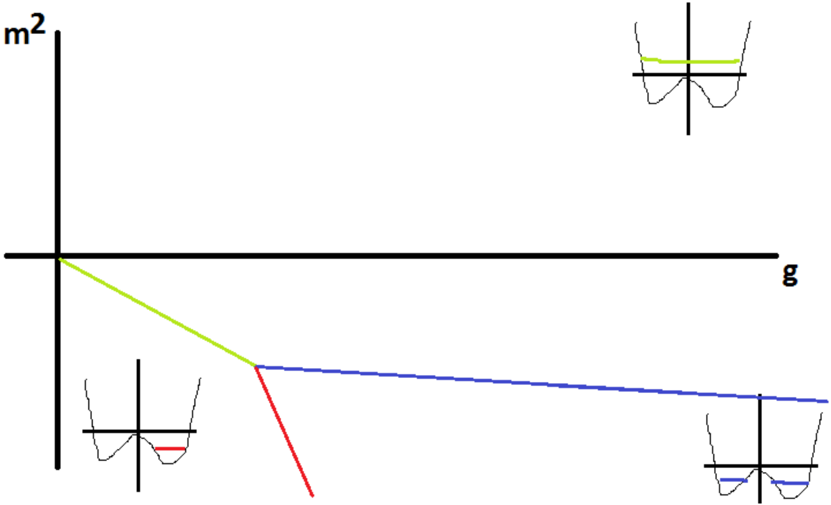
$$S[\phi] = \int d^2x \left(\frac{1}{2}\partial_i\phi\partial_i\phi + \frac{1}{2}m^2\phi^2 + g\phi^4 \right)$$



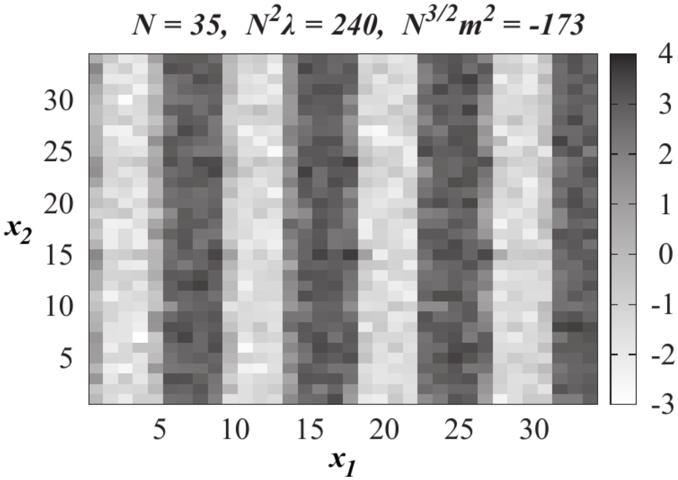
$$S[M] = \mathrm{Tr}\left(\frac{1}{2}rM^2 + gM^4\right)$$



$$S[M] = \mathrm{Tr}\left(\frac{1}{2}M[L_i,[L_i,M]] + \frac{1}{2}rM^2 + gM^4\right)$$



Mejía-Díaz, Bietenholz, Panero ’14



Introduction and outline

In this talk, I will

- briefly describe fuzzy field theories and the UV/IR mixing,
- describe fuzzy field theories in terms of a random matrix model,
- and investigate properties of models which should eventually describe a theory without the UV/IR mixing.

Fuzzy field theories

Scalar field theory on fuzzy sphere

- **Commutative**

$$S(\Phi) = \int dx \left[\frac{1}{2}\Phi\Delta\Phi + \frac{1}{2}m^2\Phi^2 + V(\Phi)\right]$$

$$\langle F \rangle = \frac{\int D\Phi\, F(\Phi) e^{-S(\Phi)}}{\int D\Phi\, e^{-S(\Phi)}} \, .$$

- **Noncommutative** (for S^2_F)

$$S(M) = \frac{4\pi R^2}{N} \mathrm{Tr}\left[\frac{1}{2} M \frac{1}{R^2}[L_i,[L_i,M]] + \frac{1}{2}m^2 M^2 + V(M)\right]$$

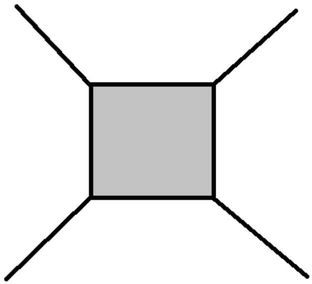
$$\langle F \rangle = \frac{\int dM\, F(M) e^{-S(M)}}{\int dM\, e^{-S(M)}} \, .$$

Grosse, Klimčík, Prešnajder '90s

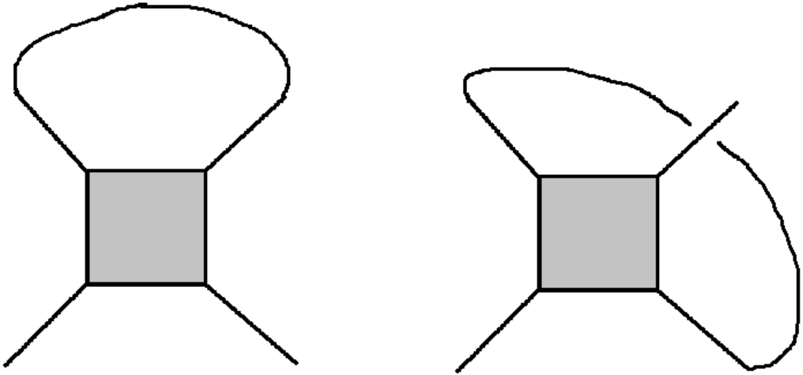
Balachandran, Kürkçüoğlu, Vaidya '05; Szabo '03

Scalar field theory on fuzzy sphere

$$M = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} c_{lm} T_{lm}$$

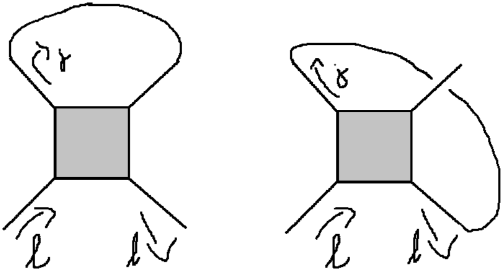


Scalar field theory on fuzzy sphere



UV/IR mixing

UV/IR on fuzzy sphere, Chu, Madore, Steinacker '01



$$I^P = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2}$$

$$I^{NP} = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2} (-1)^{l+j+N-1} \left\{ \begin{array}{ccc} l & s & s \\ j & s & s \end{array} \right\} , \; s = \frac{N-1}{2}$$

UV/IR on fuzzy sphere, Chu, Madore, Steinacker '01

$$I^{NP} - I^{P} = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2} \left[(-1)^{l+j+N-1} \begin{Bmatrix} l & s & s \\ j & s & s \end{Bmatrix} - 1 \right]$$

- This difference is finite in $N \to \infty$ limit.
- One can get quite far for small l.
- $N \to \infty$ limit of the effective action is different from the standard S^2 effective action.
- In the planar limit $S^2 \to \mathbb{R}^2$ one recovers singularities and the standard UV/IR-mixing.

Removal of UV/IR mixing on the fuzzy sphere

Removal of UV/IR mixing on S_F^2, Dolan, O'Connor, Prešnajder '01

- These problems are genuine for the two point functions and there is no such anomaly in coupling renormalization.
- By properly modifying the kinetic term of the original naive theory one can subtract the problematic anomalous term

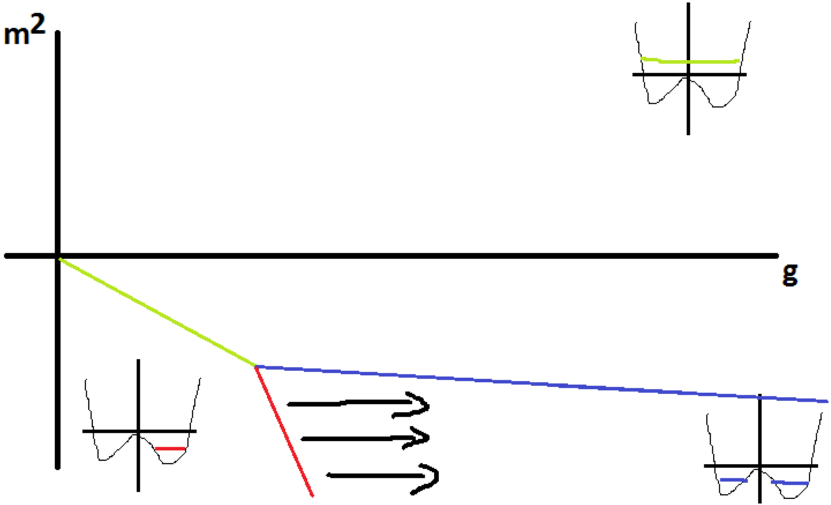
$$S = \mathrm{Tr}\left(\frac{1}{2}M[L_i,[L_i,M]] + 12gMQM + \frac{1}{2}m^2M + gM^4\right)$$

where

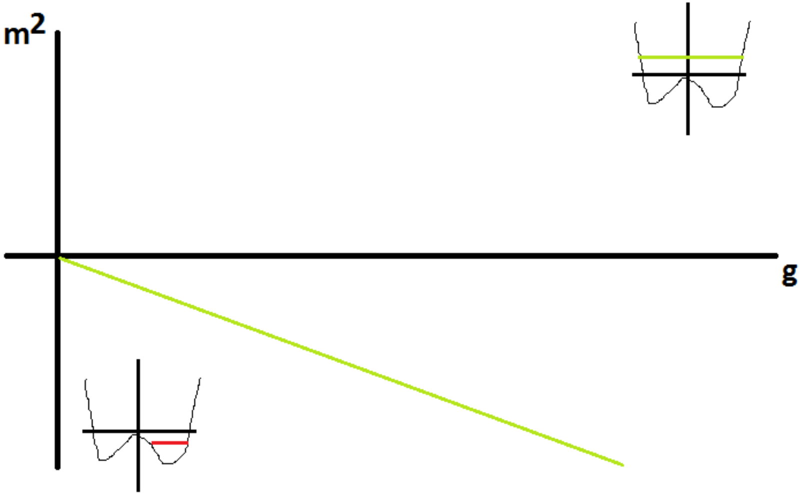
$$QT_{lm} = \underbrace{-\left(\sum_{j=0}^{N-1}\frac{2j+1}{j(j+1)+m^2}\left[(-1)^{l+j+N-1}\begin{Bmatrix} l & s & s \\ j & s & s\end{Bmatrix} - 1\right]\right)}_{Q(l)} T_{lm}\ .$$

- How does the phase diagram of this theory look?

Removal of UV/IR mixing on S_F^2



Removal of UV/IR mixing on S_F^2



Second moment multitrace matrix model for fuzzy field theory

Matrix models

- Ensemble of hermitian $N \times N$ matrices with a probability measure $S(M)$ and expectation values
$$\langle F \rangle = \frac{\int dM\, F(M) e^{-S(M)}}{\int dM\, e^{-S(M)}} \,.$$
- This is the very same expression as for the real scalar field.
- Fuzzy field theory = matrix model with

$$S(M) = \frac{1}{2} \mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2} r\, \mathrm{Tr}\left(M^2\right) + g\, \mathrm{Tr}\left(M^4\right)$$

(minus the red Brezin, Itzykson, Parisi, Zuber '78)

Matrix models of fuzzy field theories

- The large N limit of the model **without** the kinetic term

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)$$

  is **well** understood.

- The key is diagonalization and the saddle point approximation.

Matrix models of fuzzy field theories

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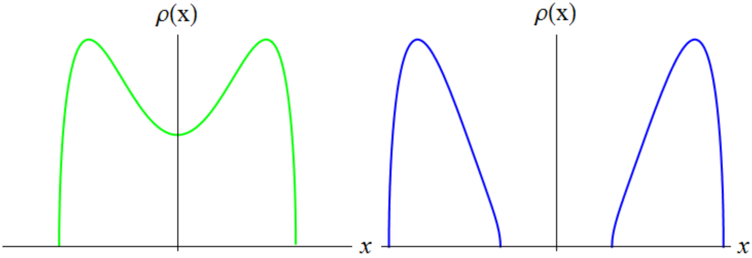
is **well** understood.

- The key results is that for $r < -4\sqrt{g}$ we get two cut eigenvalue density.



Matrix models of fuzzy field theories

- The model **with** the kinetic term

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)$$

  is **not well** understood.

  Steinacker '05; JT Acta Physica Slovaca '15

- The key issue being that diagonalization no longer straightforward.

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- The key issue being that diagonalization no longer straightforward.
- We are to compute integrals like

$$\langle F\rangle \sim \int \left(\prod_{i=1}^{N} d\lambda_i\right) F(\lambda_i)\, e^{-N^2\left[\frac{1}{2}r\frac{1}{N}\sum\lambda_i^2 + g\frac{1}{N}\sum\lambda_i^4 - \frac{2}{N^2}\sum_{i<j}\log|\lambda_i-\lambda_j|\right]}$$

$$\times \int dU\, e^{-N^2\frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger[L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

Matrix models of fuzzy field theories

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- The key issue being that diagonalization no longer straightforward.
- We are to compute integrals like

$$\langle F\rangle \sim \int \left(\prod_{i=1}^{N} d\lambda_i\right) F(\lambda_i)\, e^{-N^2\left[S_{eff}(\lambda_i)+\frac{1}{2} r \frac{1}{N}\sum \lambda_i^2 + g\frac{1}{N}\sum\lambda_i^4 - \frac{2}{N^2}\sum_{i<j}\log|\lambda_i-\lambda_j|\right]}$$

$$e^{-N^2 S_{eff}(\lambda_i)} = \int dU\, e^{-N^2 \frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger [L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

- How to compute S_{eff}?

Hermitian matrix model of fuzzy field theories

- For the free theory $g = 0$ the kinetic term just rescales the eigenvalues. Steinacker '05
- There is a unique parameter independent effective action that reconstructs this rescaling. Polychronakos '13

$$S_{eff} = \frac{1}{2} F(c_2) + \mathcal{R} = \frac{1}{2} \log \left(\frac{c_2}{1 - e^{-c_2}} \right) + \mathcal{R} \ , \ c_n = \frac{1}{N} \mathrm{Tr}\left(M^n\right)$$

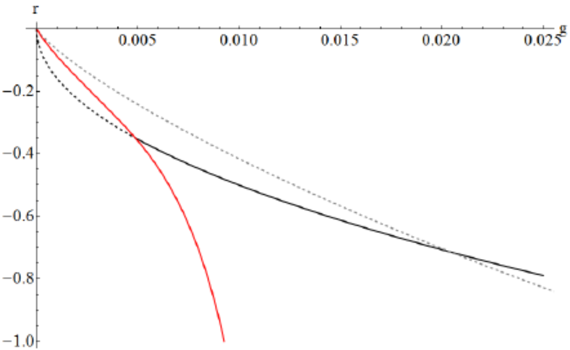
- Introducing the asymmetry $c_2 \to c_2 - c_1^2$ we obtain a matrix model

$$S(M) = \frac{1}{2} F(c_2 - c_1^2) + \frac{1}{2} r \,\mathrm{Tr}\left(M^2\right) + g \,\mathrm{Tr}\left(M^4\right) \quad , \quad F(t) = \log \left(\frac{t}{1 - e^{-t}} \right)$$

Polychronakos '13; JT '15, JT '17

Matrix models of fuzzy field theories

- Such F introduces a (not too strong) interaction among the eigenvalues. For some values of r, g an asymmetric configuration can become stable.
- It corresponds to the "standard" symmetry broken phase.



JT '17; Šubjaková, JT '19

Matrix models of fuzzy field theories

- This is result of an analytic calculation.
- A very good qualitative agreement. A very good quantitative agreement in the critical coupling.
- Different value for the critical mass parameter and different behaviour of the asymmetric transition line for large $-r$.
- We need to include $\mathcal{R}$ in a nonperturbative way.
  work in progress with M. Šubjaková

Towards a matrix model of UV/IR free theory

Towards a matrix model of UV/IR free theory

- We would like to analyze the more complicated model

$$S = \mathrm{Tr}\left(\frac{1}{2}M[L_i,[L_i,M]] + a12gMQM + \frac{1}{2}m^2M + gM^4\right)$$

where

$$QT_{lm} = \underbrace{-\left(\sum_{j=0}^{N-1}\frac{2j+1}{j(j+1)+m^2}\left[(-1)^{l+j+N-1}\left\{\begin{array}{ccc} l & s & s \\ j & s & s\end{array}\right\} - 1\right]\right)}_{Q(l)} T_{lm} \ .$$

- The previous method works for any model with a kinetic term $\mathcal{K}$, which is diagonal in T_{lm} basis

$$\mathcal{K}T_{lm} = K(l)T_{lm} \ .$$

$$K(l) = l(l+1) - a12gQ(l) \ .$$

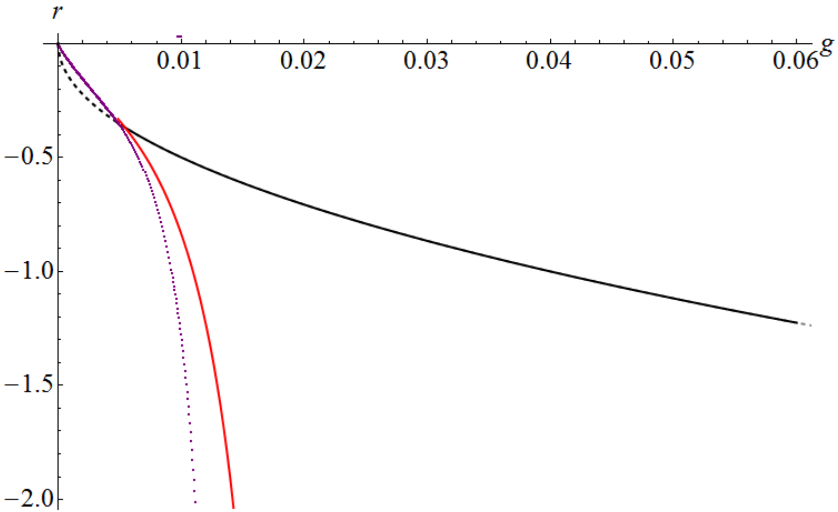
Towards a matrix model of UV/IR free theory

- Operator Q can be expressed as a power series in $C_2 = [L_i, [L_i, \cdot]]$

$$Q = q_1 C_2 + \dots$$

- As a starting point, it is interesting to see the phase structure of such simplified model.
  O'Connor, Säman '07

Towards a matrix model of UV/IR free theory



Conclusions and outlook

Conclusions

- We can achieve movement in the phase diagram by modifying the kinetic term of the theory.
- Making steps in the direction of the UV/IR free theory produces expected results.
- But there is plenty more.

Outlook

- Further analysis beyond $Q = q_1 C_2$.
- Numerical analysis of the $\mathcal{K} = C_2 + 12gQ$ model.
- What about four dimensions. Especially $\mathbb{C}P^2$. second moment approximation.

- More complete analysis of the matrix model beyond the second moment approximation.