

QUANTUM-ENHANCED HOLOMETER REALIZATION

Paolo Traina

COST ACTION MP1405 - Quantum Structure of Spacetime

Bratislava, February 11 2019



Outline

- INTRO: why correlating interferometers?
- EXPERIMENT
- RESULTS (I): independent squeezed states
- RESULTS (II): twin beam-like correlations
- Conclusions & Outlook



QUANTUM OPTICS GROP at INRIM



Siva Pradyumna, Elena Losero, Ivano Ruo Berchera, Ivo P. Degiovanni, Marco Genovese



Massimo Zucco



Stefano Olivares (Univ. Milano)



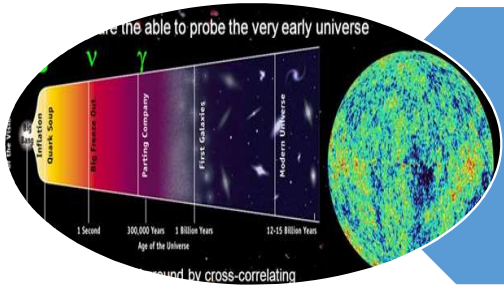
Christian S. Jacobsen

Tobias Gehring

Ulrik L. Andersen

Why correlating interferometers?

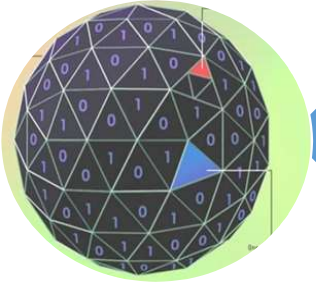
Research of stochastic signal by correlating interferometers



Stochastic Gravitational Wave Background
 (10^{-36} to 10^{-32} seconds after the Big Bang, whereas the CMB was produced approximately 300,000 years later)
 [Search for a Stochastic Background of 100-MHz Gravitational Waves with Laser Interferometers, PRL 101, 101101 (2008)]
 [Upper Limits on the Stochastic Gravitational-Wave Background from Advanced LIGO's First Observing Run. Phys. Rev. Lett., 118:121101, 2017]



Traces of primordial blackholes
 [MHz gravitational wave constraints with decameter Michelson interferometers, PRD 95, 063002 (2017)]



Fundamental noise at the plank scale in quantum gravity model
 [First Measurements of High Frequency Cross-Spectra from a Pair of Large Michelson Interferometers, PRL 117, 111102 (2016)]
 [PRD 85, 064007 (2012)]
 [Models of exotic interferometer cross-correlations in emergent space-time. Class. and Quantum Grav., 35(20), 204001 (2018)]

Research of stochastic signal by correlating interferometers

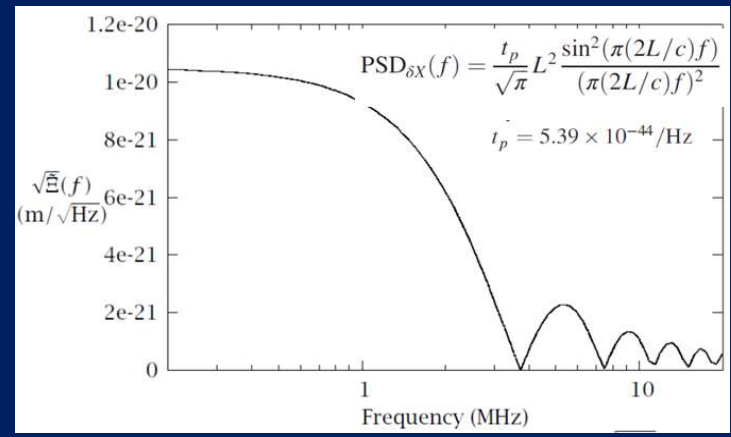
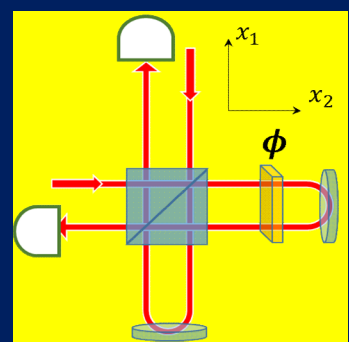
Heuristic QG theories predicts non-commutativity of position/rotational variables at Planck scale

$$[\hat{x}_i, \hat{x}_j] = \hat{x}_k \epsilon_{ijk} i c t_P / \sqrt{4\pi}$$



Fundamental space-time uncertainty principle (Holographic Noise)

Likely detectable in Michelson interferometer



Fundamental scale in quantum gravity model

[First Measurements of High-Frequency Cross-Spectra from a Pair of Large Michelson Interferometers. Phys. Rev. Lett. 117, 111102 (2016)]

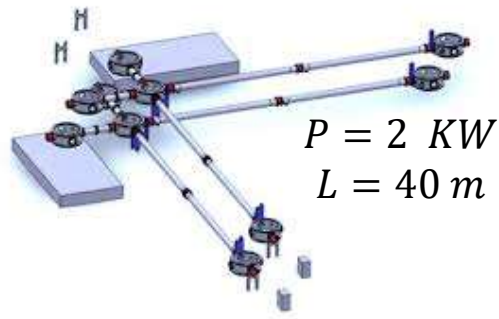
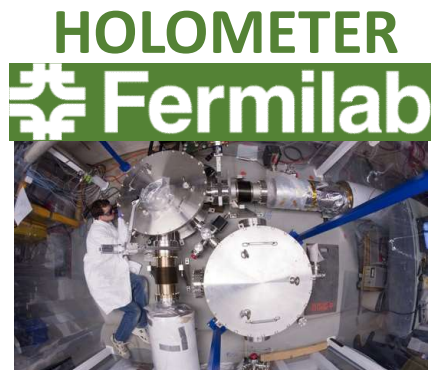
[PRD 85, 063002 (2012)]

[Models of exotic interferometer cross-correlations in emergent space-time. Class. and Quantum Grav., 35(20), 204001 (2018)]

and
 approximately 300,000
 interferometers, PRL 101,
 First Observing Run.

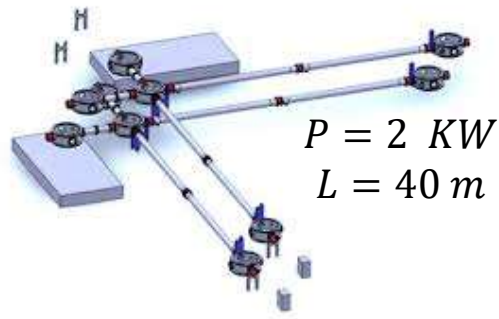
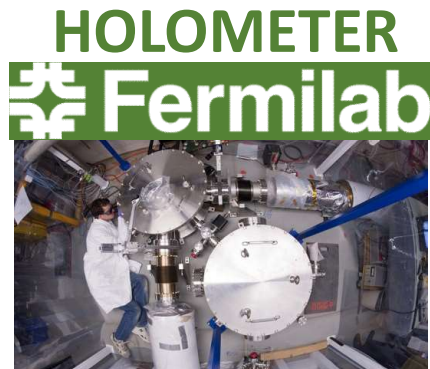
[PRD 95, 063002 (2017)]

**First Measurements of High Frequency Cross-Spectra
from a Pair of Large Michelson Interferometers**



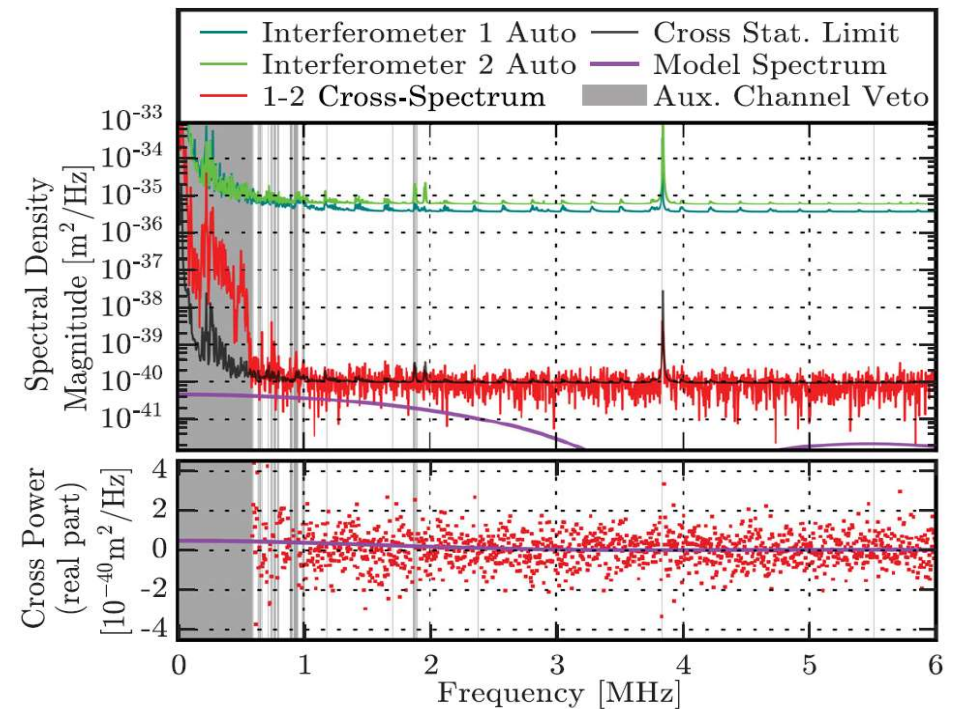
Even if the HN is hidden by the photon shot noise in one interferometer, it could emerge in the cross-correlation between two of them, if they are in the same space time volume (waiting longer enough..)

**First Measurements of High Frequency Cross-Spectra
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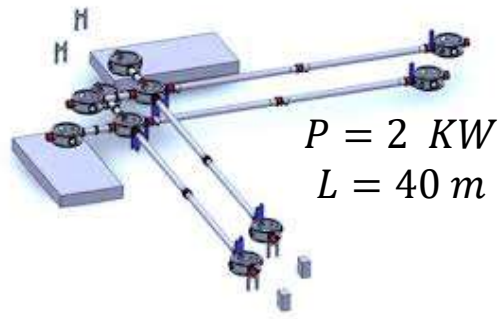
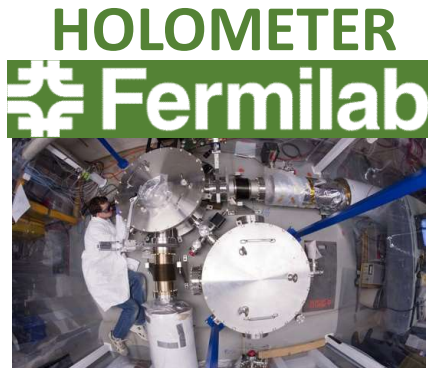


Even if the HN is hidden by the photon shot noise in one interferometer, it could emerge in the cross-correlation between two of them, if they are in the same space time volume (waiting longer enough..)

HN lower bounded to $10^{-21} \text{ m}/\sqrt{\text{Hz}}$ in the MHz region of the spectrum after 165 h of acquisition.



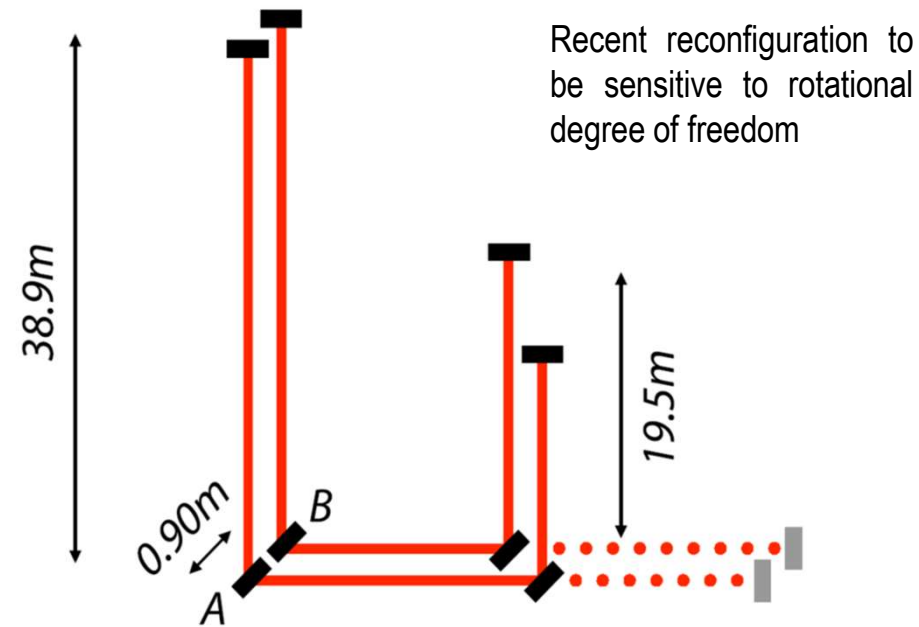
**First Measurements of High Frequency Cross-Spectra
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Even if the HN is hidden by the photon shot noise in one interferometer, it could emerge in the cross-correlation between two of them, if they are in the same space time volume (waiting longer enough..)

**Models of exotic interferometer
cross-correlations in emergent space-time**

Craig Hogan^{1,2} and Ohkyung Kwon^{2,3,4}



Quantization of the Electromagnetic Field

<i>Classical</i>		<i>Quantum</i>
$\mathbf{E}(\mathbf{r}, t) = \sum_{\mathbf{k}} \hat{\mathbf{e}}_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} \alpha_{\mathbf{k}} e^{-i\nu_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.},$		$\mathbf{E}(\mathbf{r}, t) = \sum_{\mathbf{k}} \hat{\mathbf{e}}_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} a_{\mathbf{k}} e^{-i\nu_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{r}} + \text{H.c.},$
$\mathbf{H}(\mathbf{r}, t) = \frac{1}{\mu_0} \sum_{\mathbf{k}} \frac{\mathbf{k} \times \hat{\mathbf{e}}_{\mathbf{k}}}{\nu_{\mathbf{k}}} \mathcal{E}_{\mathbf{k}} \alpha_{\mathbf{k}} e^{-i\nu_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.}$		$\mathbf{H}(\mathbf{r}, t) = \frac{1}{\mu_0} \sum_{\mathbf{k}} \frac{\mathbf{k} \times \hat{\mathbf{e}}_{\mathbf{k}}}{\nu_{\mathbf{k}}} \mathcal{E}_{\mathbf{k}} a_{\mathbf{k}} e^{-i\nu_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{r}} + \text{H.c.}$
$\alpha_{\mathbf{k}} \quad \alpha_{\mathbf{k}}^*$		$a_{\mathbf{k}} \quad a_{\mathbf{k}}^\dagger$
Unitless Coefficients		Quantum Operators
		$[a_{\mathbf{k}}, a_{\mathbf{k}}^\dagger] = 1$

Energy of a single mode quantum EM field

$$\mathcal{H}_{\mathbf{k}} = \hbar \nu_{\mathbf{k}} \left(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2} \right)$$

$$\mathcal{H}_{\mathbf{k}} |n_{\mathbf{k}}\rangle = \hbar \nu_{\mathbf{k}} \left(n_{\mathbf{k}} + \frac{1}{2} \right) |n_{\mathbf{k}}\rangle$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$



Quadrature Operators

$$X_1 = \frac{1}{2}(a + a^\dagger) \quad \text{“Amplitude” or “Position”}$$

$$X_2 = \frac{1}{2i}(a - a^\dagger) \quad \text{“Phase” or “Momentum”}$$

$$[X_1, X_2] = \frac{i}{2} \quad \longrightarrow \quad \Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

Heisenberg's Unc. Relation



Coherent States

Coherent State: eigenstate of the annihilation operator

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

Displacement operator: $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$D^{-1}(\alpha)aD(\alpha) = a + \alpha$$

Mean photon number: $\langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$

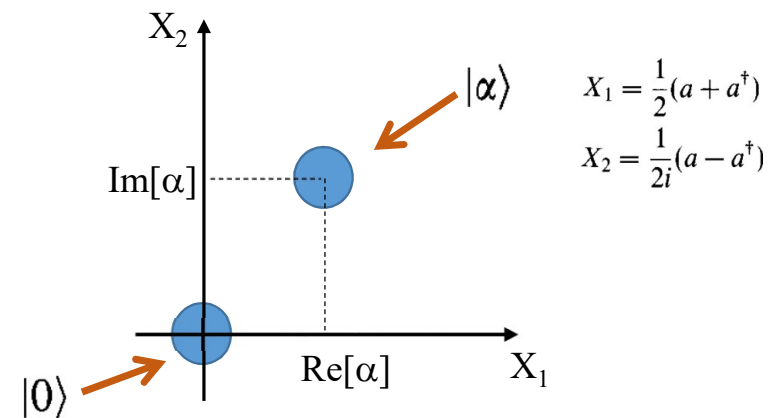
Photon number statistics: $p(n) = \langle n|\alpha\rangle\langle\alpha|n\rangle = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$ $\langle n\rangle = |\alpha|^2$

Quadrature operators

$$(\Delta X_1)^2 = \langle\alpha|X_1^2|\alpha\rangle - (\langle\alpha|X_1|\alpha\rangle)^2 = \frac{1}{4}$$

$$(\Delta X_2)^2 = \frac{1}{4}$$

$$\Delta X_1 \Delta X_2 = \frac{1}{4}$$



Squeezed States

Hamiltonian of a degenerate parametric process: $\mathcal{H} = i\hbar (ga^{\dagger 2} - g^* a^2)$

(Unitary) "Squeeze" Operator: $S(\xi) = \exp\left(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}\right)$ $\xi = r \exp(i\theta)$

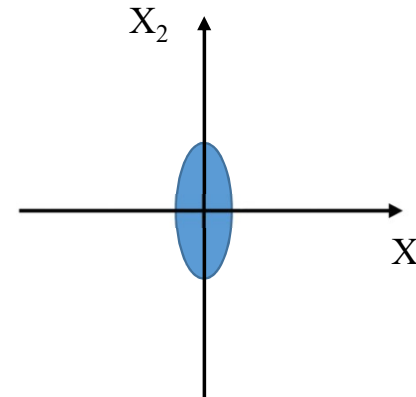
$$\begin{cases} S^\dagger(\xi)aS(\xi) = a \cosh r - a^\dagger e^{i\theta} \sinh r \\ S^\dagger(\xi)a^\dagger S(\xi) = a^\dagger \cosh r - a e^{-i\theta} \sinh r \end{cases}$$

Squeezed Vacuum: $|\xi\rangle = S(\xi)|0\rangle$

$$(\Delta X_1)^2 = \frac{1}{4}e^{-2r}$$

$$(\Delta X_2)^2 = \frac{1}{4}e^{2r}$$

$$\Delta X_1 \Delta X_2 = \frac{1}{4}$$



$$\begin{aligned} X_1 &= \frac{1}{2}(a + a^\dagger) \\ X_2 &= \frac{1}{2i}(a - a^\dagger) \end{aligned}$$

Squeezed Vacuum can be obtained with an OPO operating under threshold



Phase measurement in an interferometer

The input-output relations of the mode operators of an interferometer are the same of a BS with T (given by the phase ϕ_p)

- $|0\rangle$ in a -port, $|\alpha\rangle$ in b -port

$$\langle n_{cd} \rangle = |\alpha|^2 \cos(\phi_p)$$

$$(\Delta n_{cd})^2 = |\alpha|^2$$

$$\Delta\phi = \frac{\Delta n_{cd}}{|\partial \langle n_{cd} \rangle / \partial \phi_p|} = \frac{1}{\sqrt{\langle n \rangle}} \quad \leftarrow \text{Shot-Noise Limit}$$

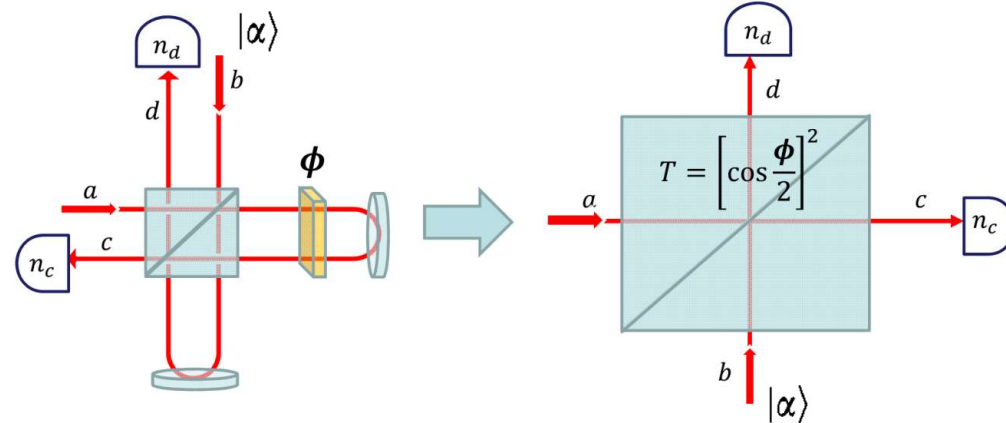
$$\langle n \rangle = |\alpha|^2$$

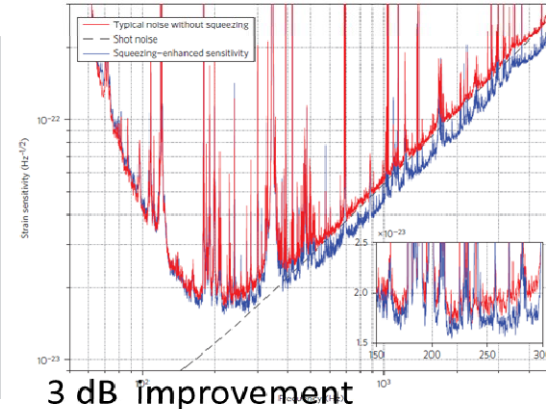
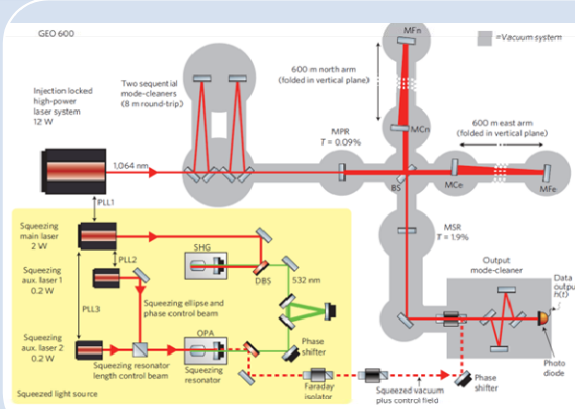
- $|\xi\rangle$ in a -port, $|\alpha\rangle$ in b -port ($\theta = 2\phi_l$)

$$\langle n_{cd} \rangle = (\langle n \rangle + \sinh^2 r) \cos \phi_p \cong \langle n \rangle \cos \phi_p$$

$$(\Delta n_{cd})^2 = \langle n \rangle e^{-2r} + \sinh^2 r$$

$$\Delta\phi = \frac{\Delta n_{cd}}{|\partial \langle n_{cd} \rangle / \partial \phi_p|} = \frac{e^{-r}}{\sqrt{\langle n \rangle}} \quad \leftarrow \text{Below the Shot-Noise Limit}$$





Squeezed light in gravitational wave detectors

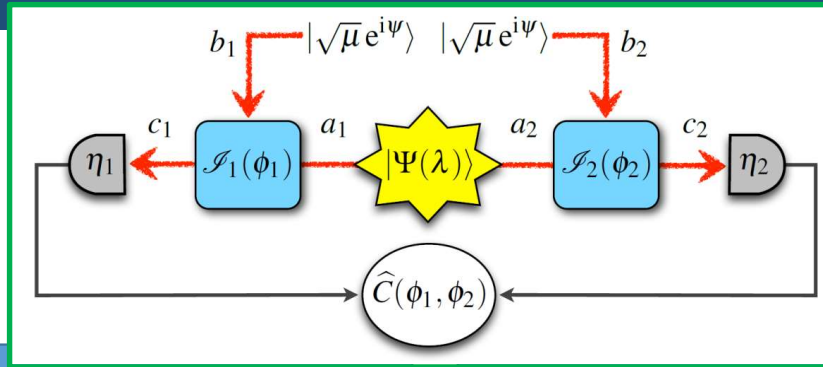
A sub-shot noise PS measurement in a **single** interferometer (e.g. gravitational wave detector) was suggested exploiting squeezed light

[Caves, PRD **23**, 1693 (1981), Kimble et al., PRD **65**, 022002 (2001)]

...

and recently realized at Geo 600 and LIGO

[R. Schnabel et al., Nature Commun. **1**, 121 (2010), Ligo, Nature Phys. **7**, 962 (2011)]



Two independent squeezed states

$$|\Psi(\lambda)\rangle_{a_1, a_2} = S_{a_1}(\xi) S_{a_2}(\xi) |0\rangle_{a_1} \otimes |0\rangle_{a_2}$$

Quadrature squeezing

$$\Delta^2 X = \frac{e^{-2r}}{2} \quad \longrightarrow \quad \langle \Delta^2 \phi \rangle_{SQ} = \langle \Delta^2 \phi \rangle_{SNL} \left(e^{-2r} + \frac{1-\eta}{\eta} \right)$$

Caves, PRD 23, 1693 (1981)

Photon noise reduction in each interferometer helps in the detection of correlated signals

$$\langle \Delta^2(\Delta\phi_1 \Delta\phi_2) \rangle_{SQ \times SQ} < \langle \Delta^2(\Delta\phi_1 \Delta\phi_2) \rangle_{SNL}$$

Two mode squeezing (Twin Beam)

$$|\Psi(\lambda)\rangle_{a_1, a_2} = \frac{1}{\sqrt{1+\lambda}} \sum_{m=0}^{\infty} \left(e^{i\theta} \sqrt{\frac{\lambda}{1+\lambda}} \right)^m |m, m\rangle_{a_1, a_2}$$

- Quadrature correlations:

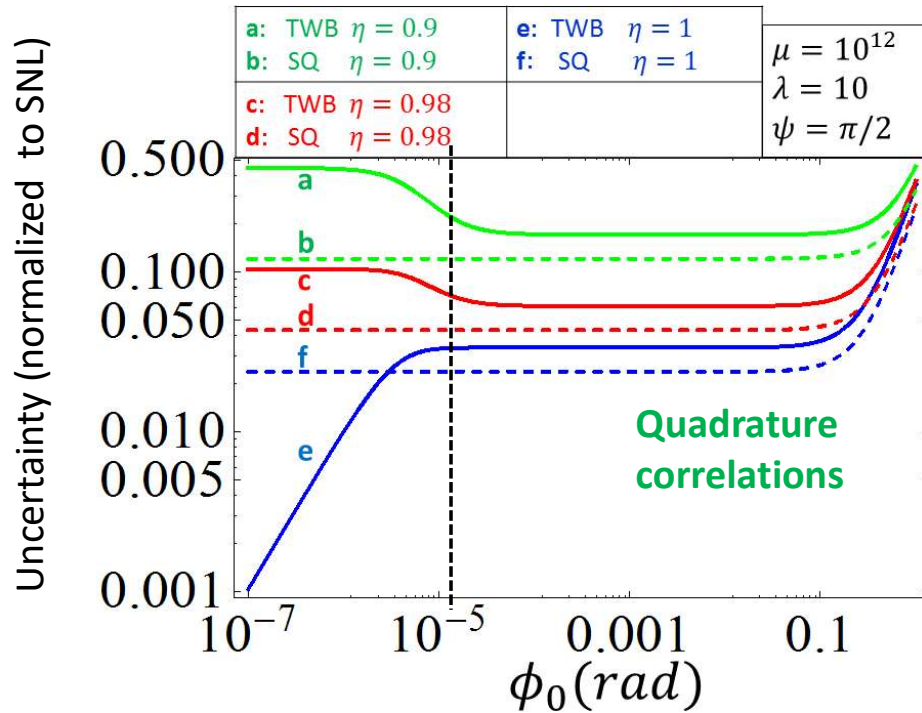
$$\Delta^2(X_1 - X_2) = \frac{e^{-2r}}{2} \quad \Delta^2(Y_1 + Y_2) = \frac{e^{-2r}}{2}$$

- Photon number ENTANGLEMENT

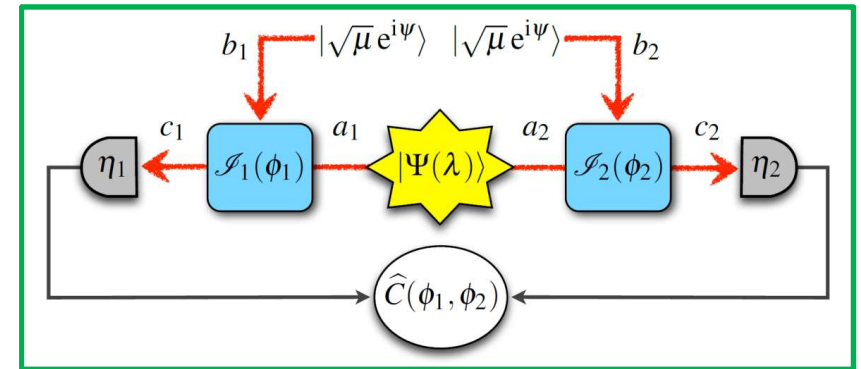
$${}_{a_1, a_2} \langle \Psi(\lambda) | (m_1 - m_2)^M | \Psi(\lambda) \rangle_{a_1, a_2} = 0$$

$$\Delta^2(N_1 - N_2) = 0$$

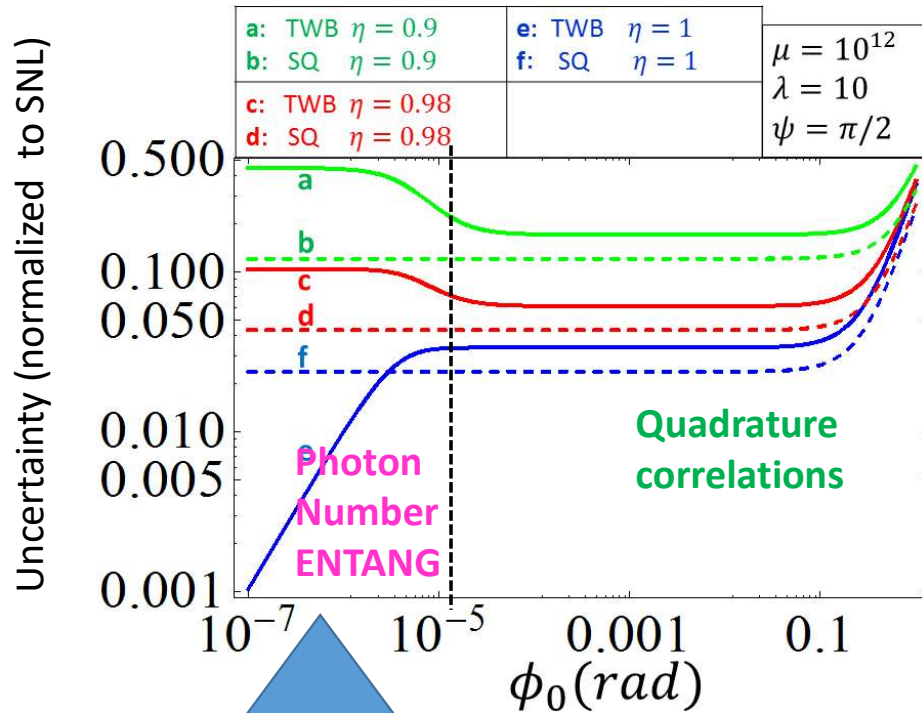
PRL 110, 213601 (2013), PRA 92, 053821 (2015)



- ϕ_0 central working phase
- η detection efficiency
- λ number of photon of quantum light
- μ number of photon of coherent state

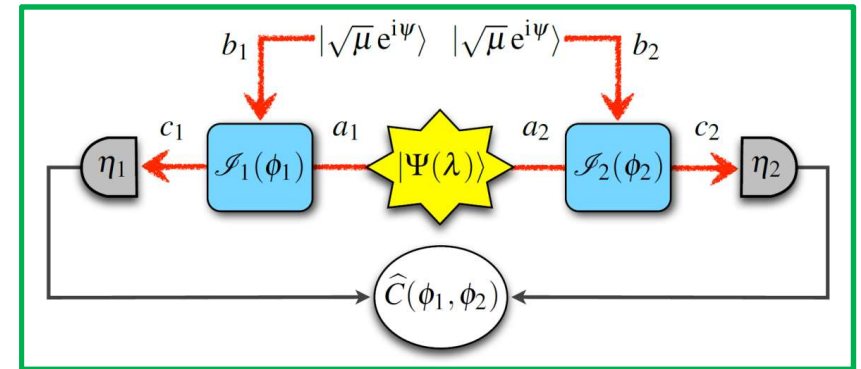


PRL **110**, 213601 (2013), PRA **92**, 053821 (2015)

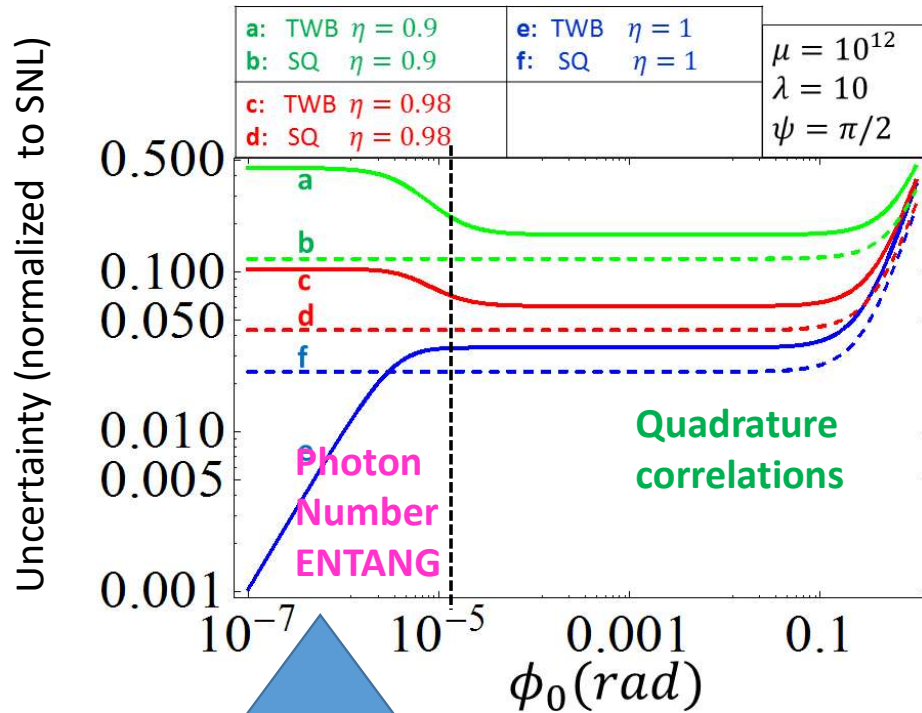


TWB is transmitted at the read-out ports while coherent is completely reflected
 $\tau = (\cos \phi/2)^2 \approx 1$ $\mu(1 - \tau)/\tau\lambda \ll 1$

- ϕ_0 central working phase
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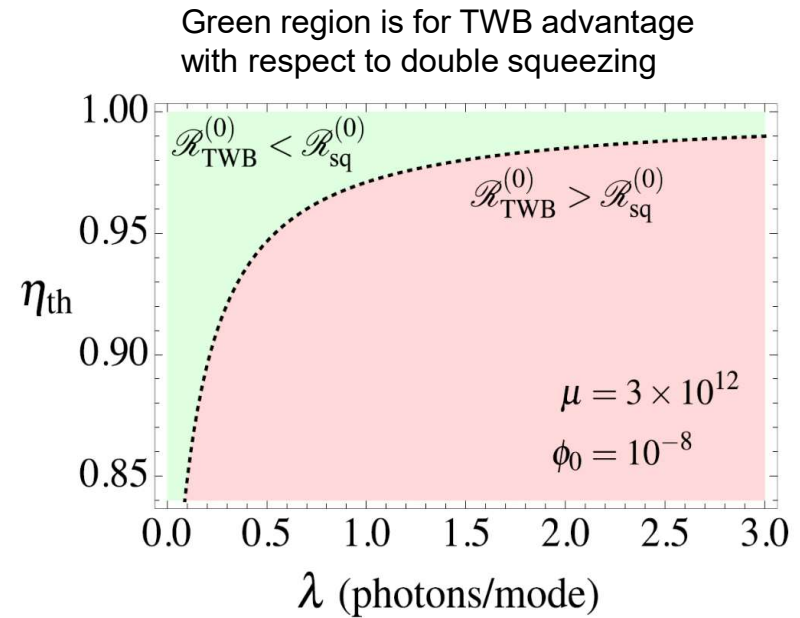


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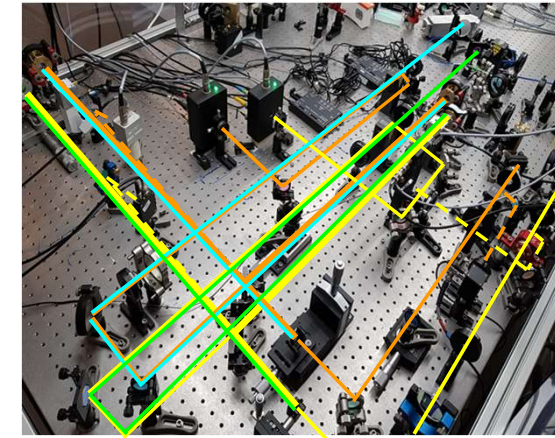
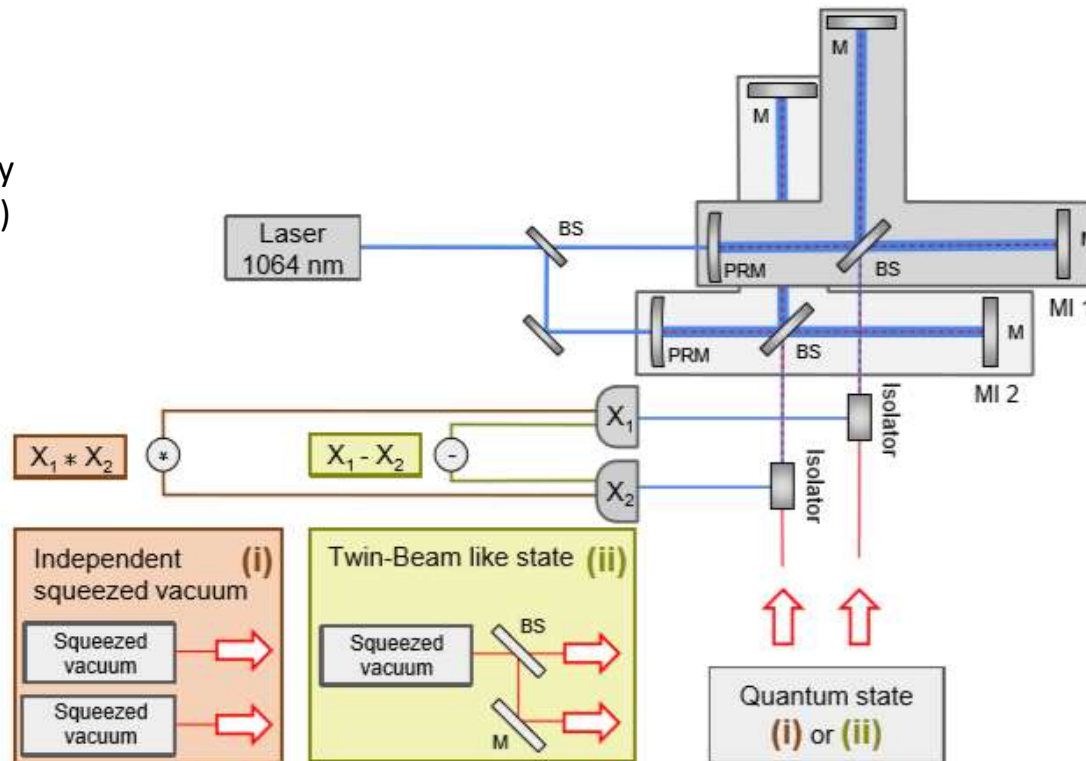
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PRL **110**, 213601 (2013), PRA **92**, 053821 (2015)

EXPERIMENT

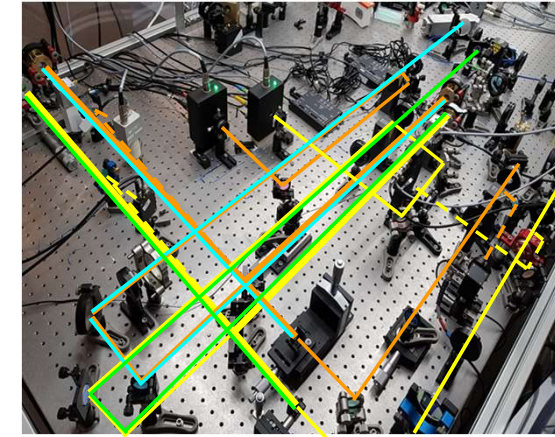
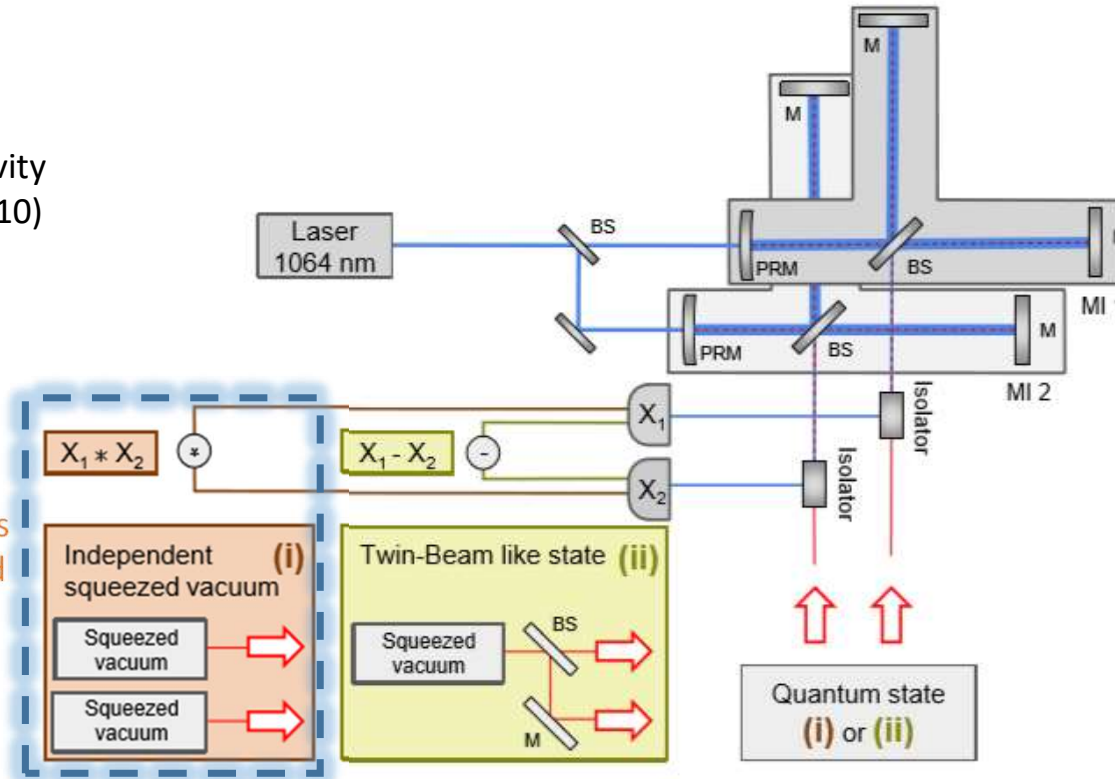
- Read-out AS port operated close to the dark fringe (LIGO, HOLOMETER)
- 2-D Power recycling cavity 90% reflectivity (gain =10)



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Alternatively:

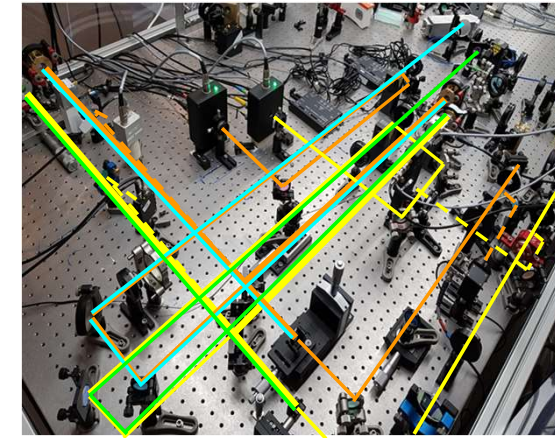
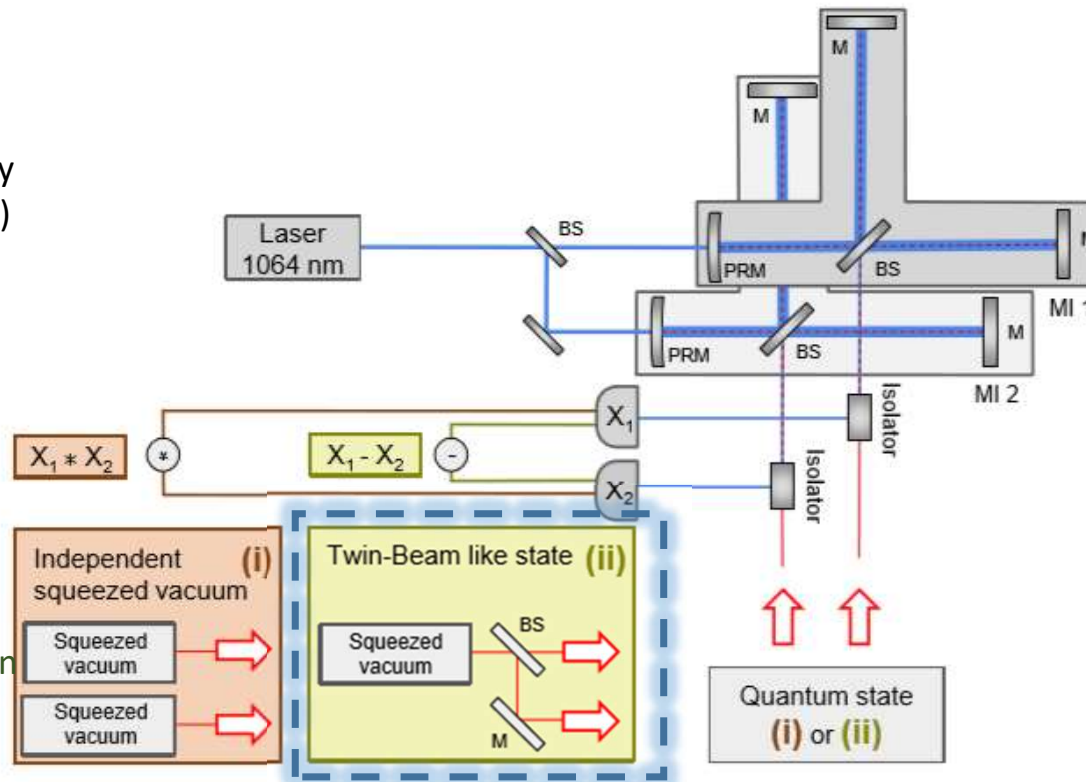
- Two independent squeezing are injected
- Cross spectrum or cross correlation is measured



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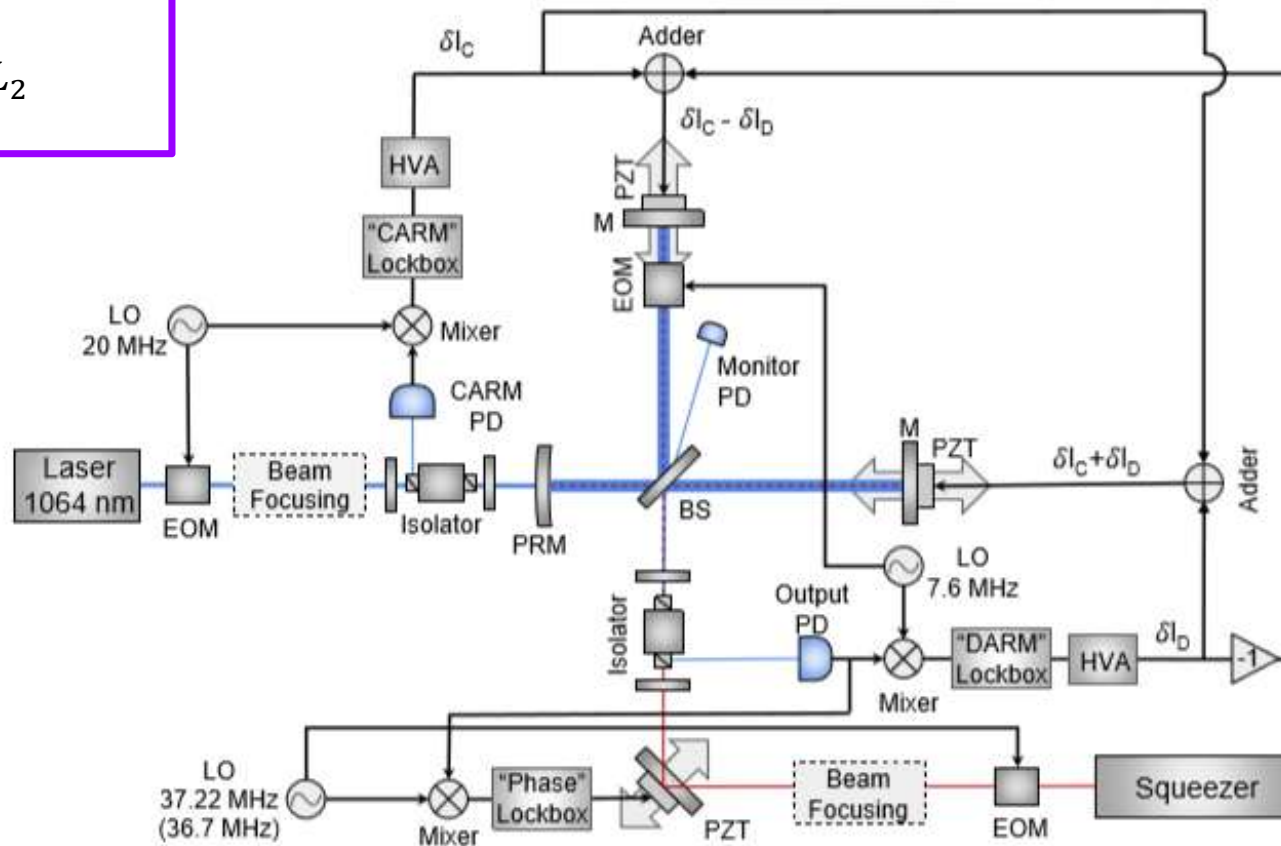
Alternatively:

- Two independent squeezing are injected
- Cross spectrum or cross correlation is measured
- Twin beam like correlation are injected
- The output difference is measured

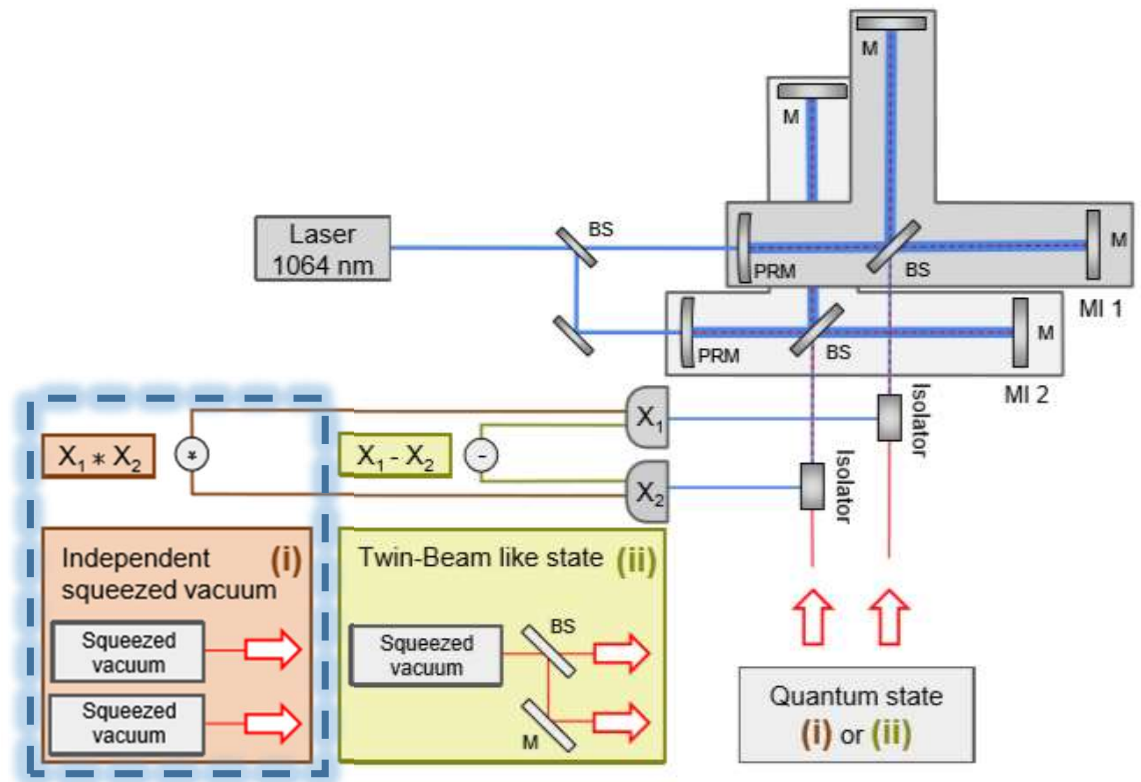
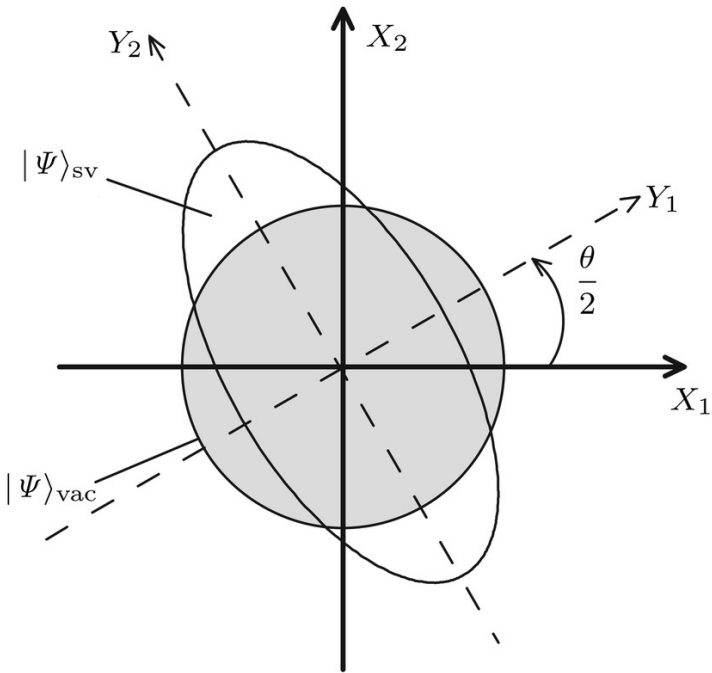


$$\text{CARM} \rightarrow \frac{L_1 + L_2}{2}$$

$$\text{DARM} \rightarrow L_1 - L_2$$

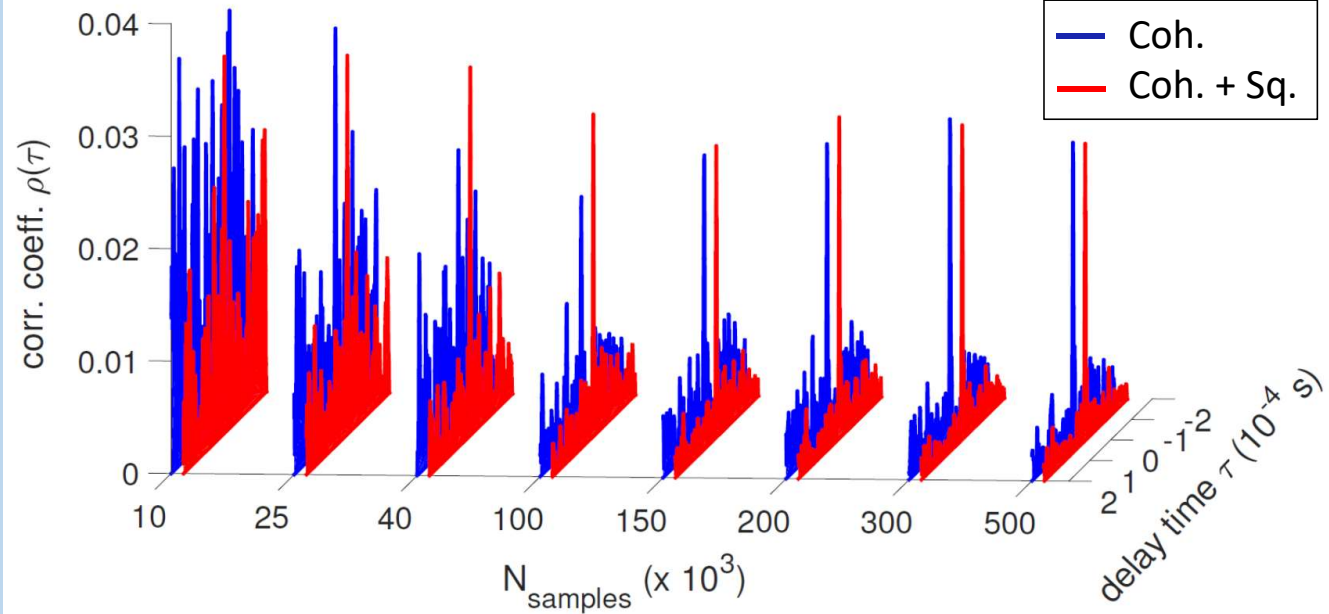


RESULTS (I): Independent squeezed states



Temporal Cross-correlation of the interferometers (SQxSQ)

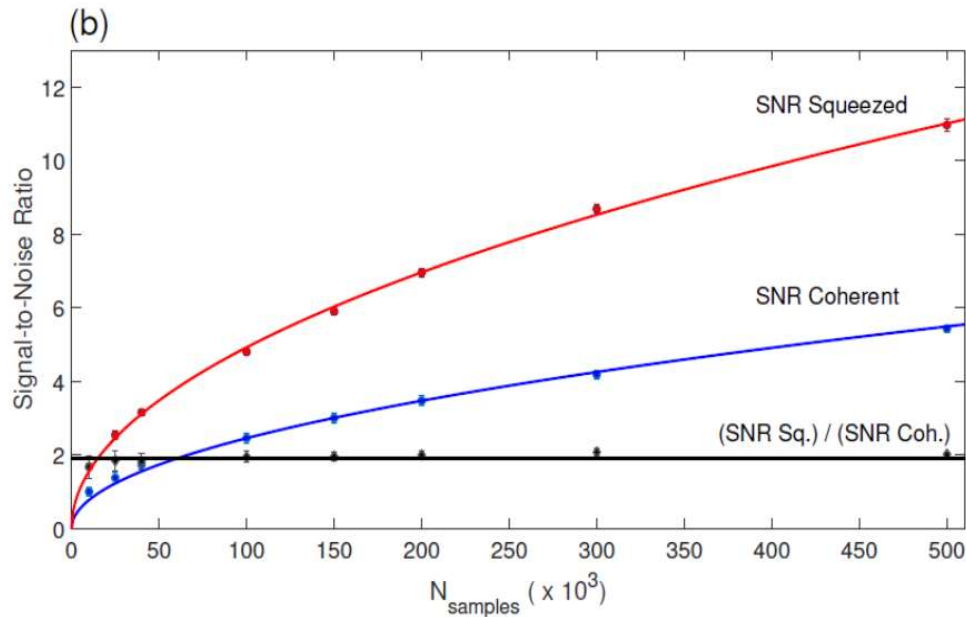
$$\rho(\tau) = \frac{|\text{Cov}(I_1(t)I_2(t + \tau))|}{\sqrt{\text{Var}(I_1(t))\text{Var}(I_2(t))}}$$



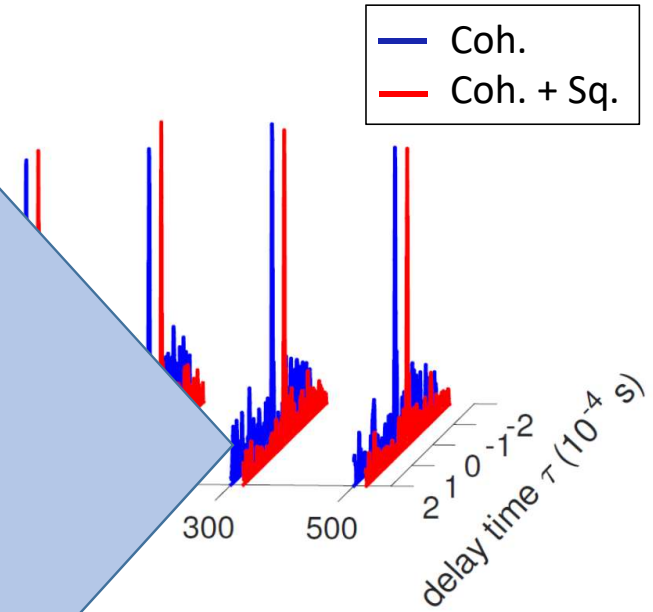
- Correlated white noise injected (about 1/5 of the shot noise level)
 - About 3dB of squeezing in each interferometer
- The cross correlation peak emerges at the increasing of the measurement time (number of samples)
 - Noise floor halved by SQ injection

[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph]

Temporal Cross-correlation of the interferometers (SQxSQ)



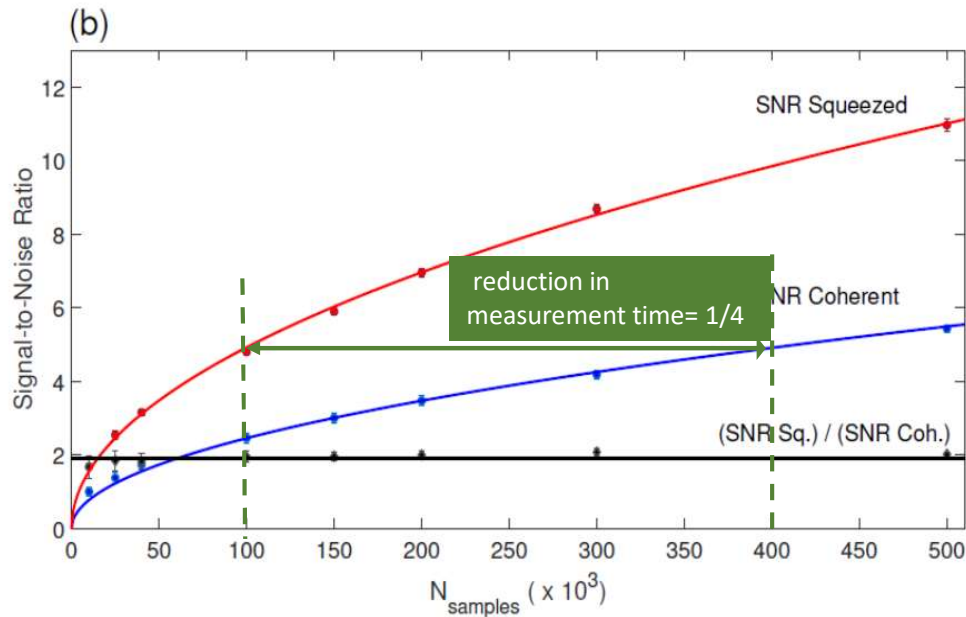
- SNR improves with the usual statistical scaling $\sqrt{N_{\text{sample}}}$
- SNR is twice when squeezing is injected



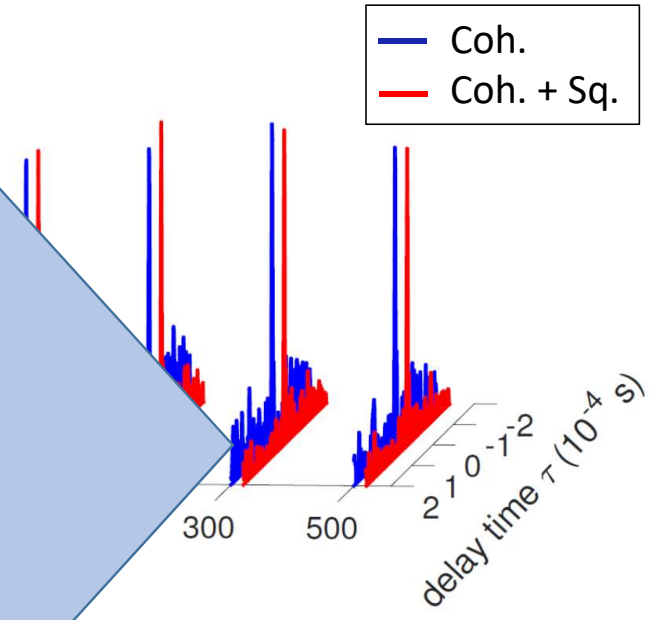
- SNR is halved (about 1/5 of the shot noise level) when squeezing is injected in each interferometer
- SNR improves and noise floor merges at the increasing of the measurement time (number of samples)
- Noise floor halved by SQ injection

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Temporal Cross-correlation of the interferometers (SQxSQ)



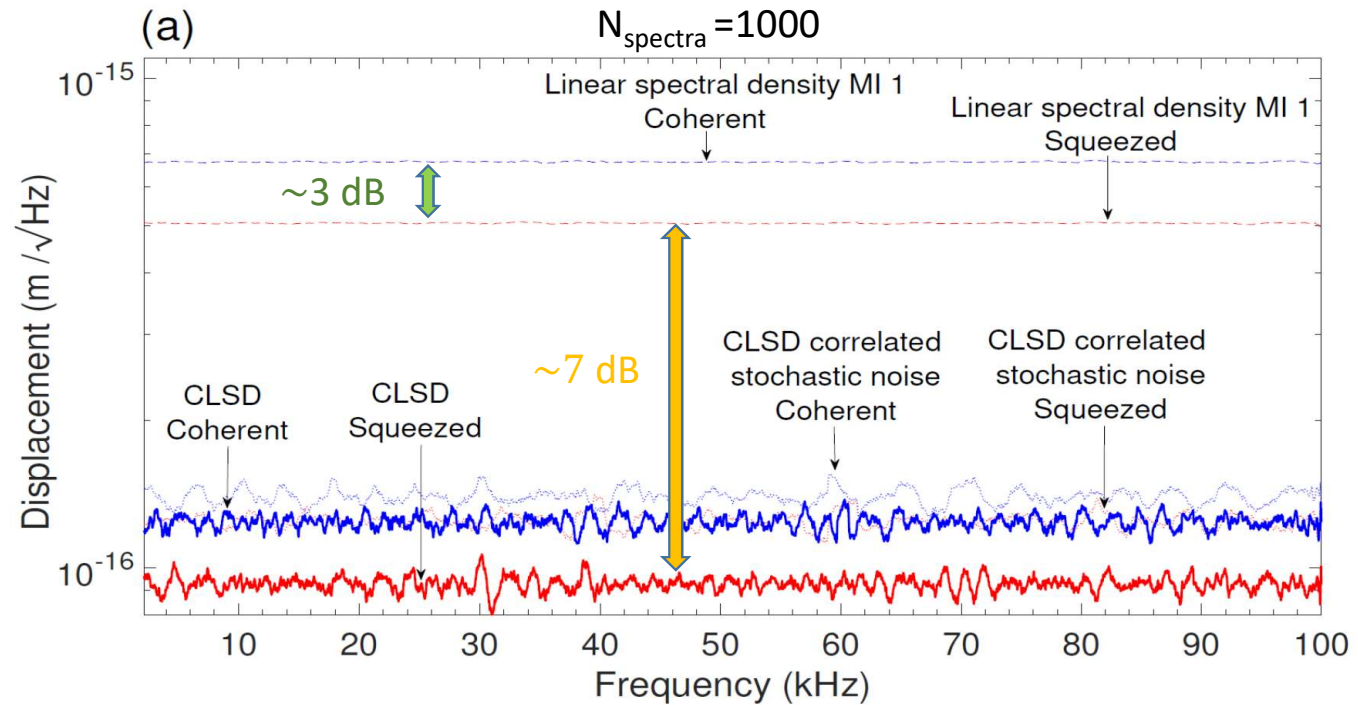
- SNR improves with the usual statistical scaling $\sqrt{N_{\text{sample}}}$
- SNR is twice when squeezing is injected
- **4 times reduction in the measurement time demonstrated**



- SNR is halved (about 1/5 of the shot noise level)
- Squeezing in each interferometer
- SNR merges at the increasing of the measurement time (number of samples)
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Cross- Linear Spectral Density (SQxSQ)

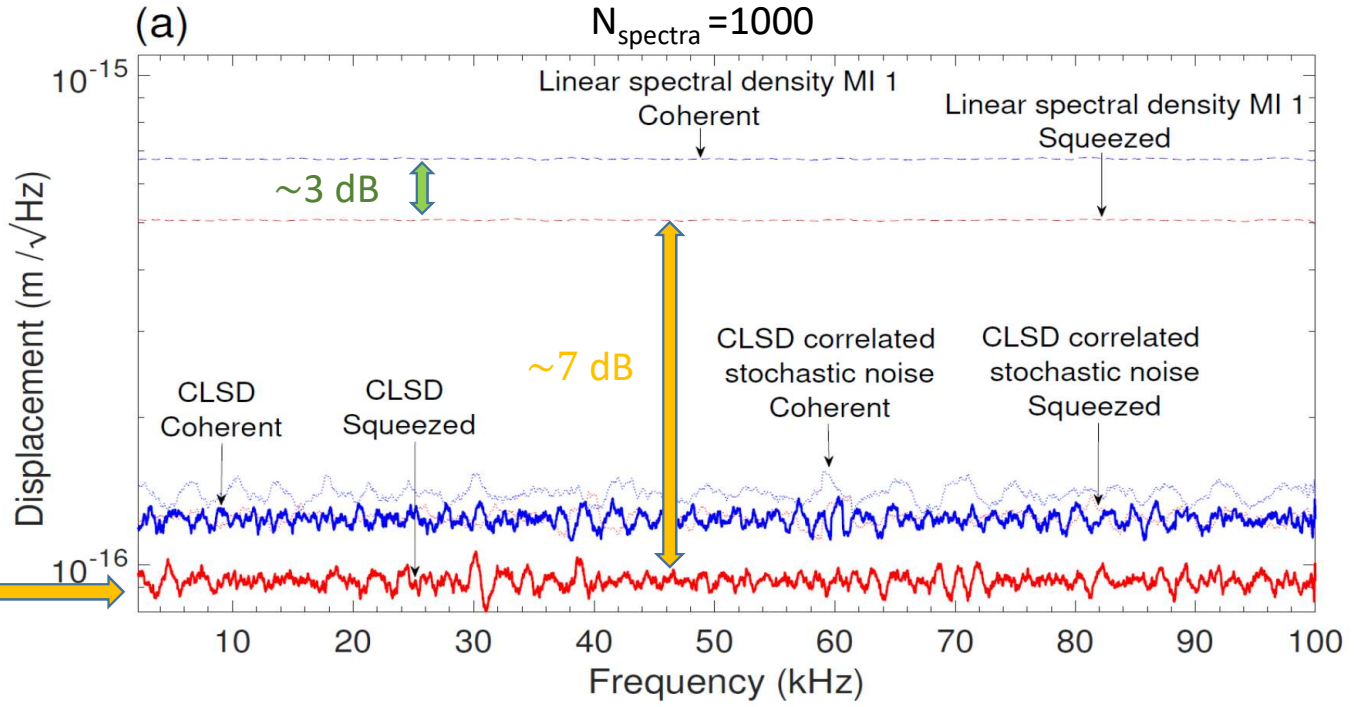


- The C-LSDs are evaluated in a BW=100KHz down-mixed at 13.5 MHz

[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph]

Cross- Linear Spectral Density (SQxSQ)

Sensitivity to correlated signals



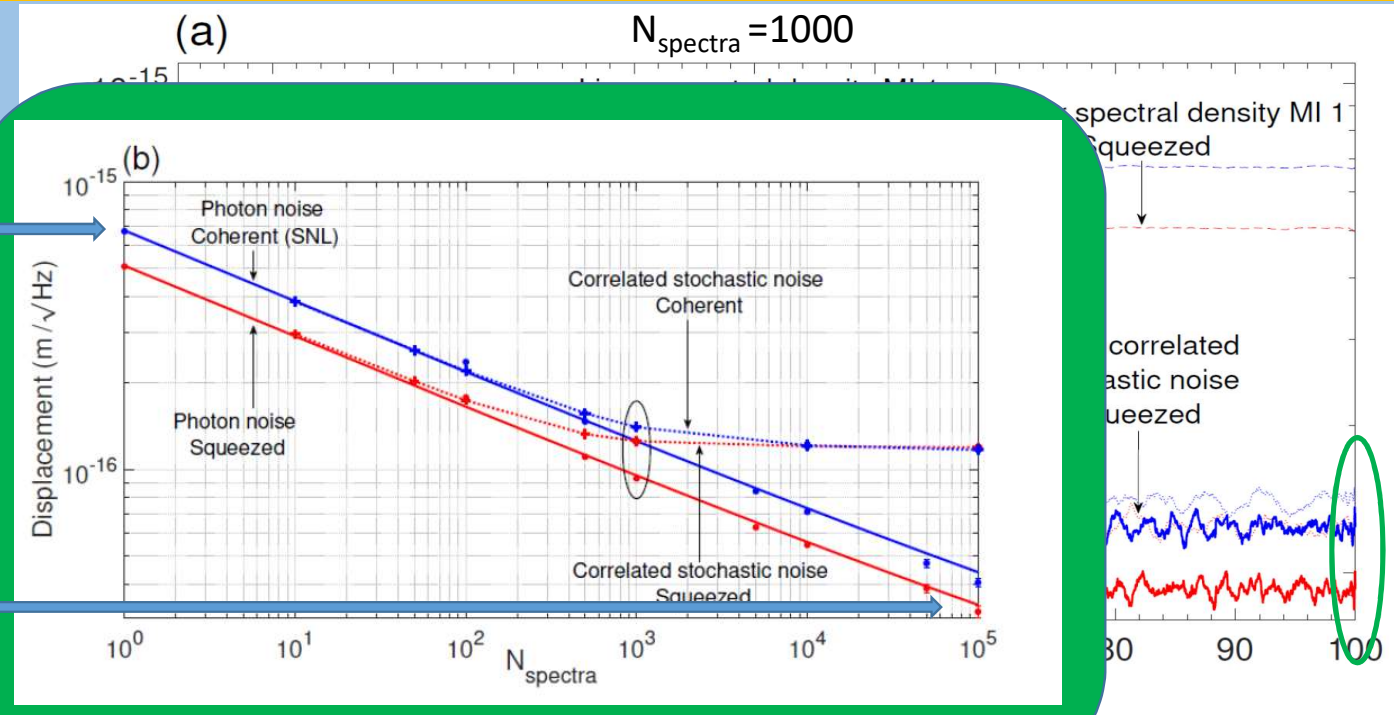
- The C-LSDs are evaluated in a BW=100KHz down-mixed at 13.5 MHz
- A correlated noise is injected (amplitude around 1/5 of the photon shot noise of each interf.)

[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph]

Cross- Linear Spectral Density (SQxSQ)

SNL Single

SNL/20



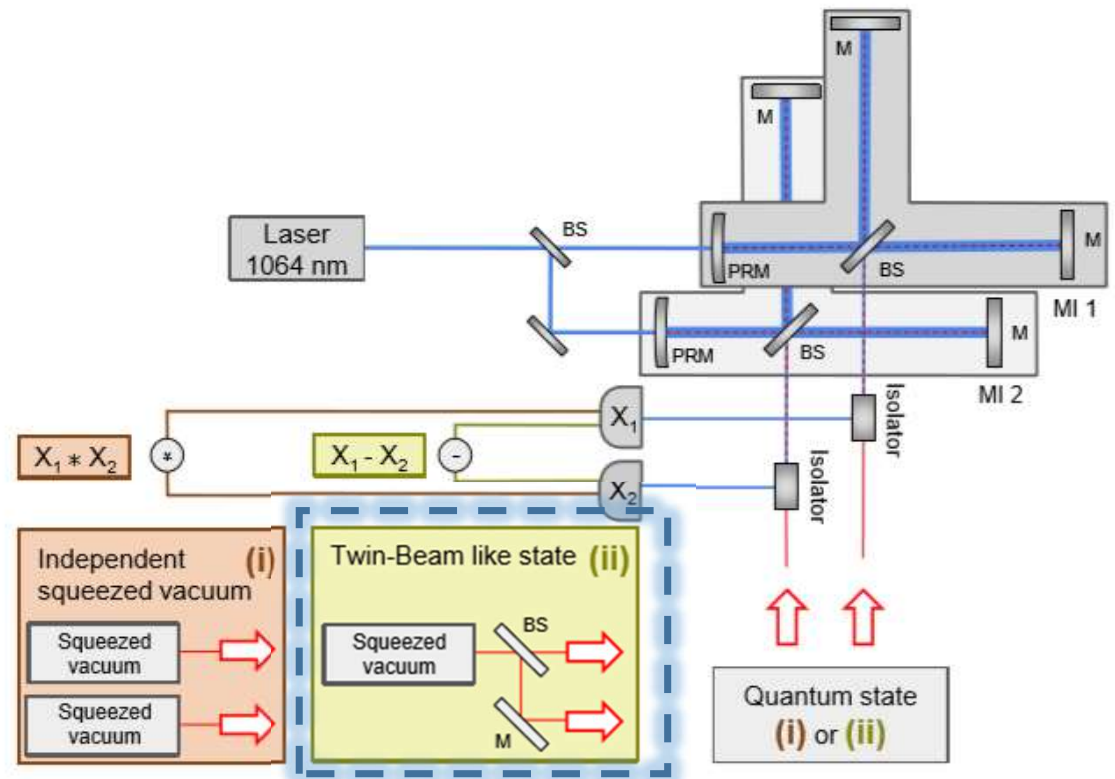
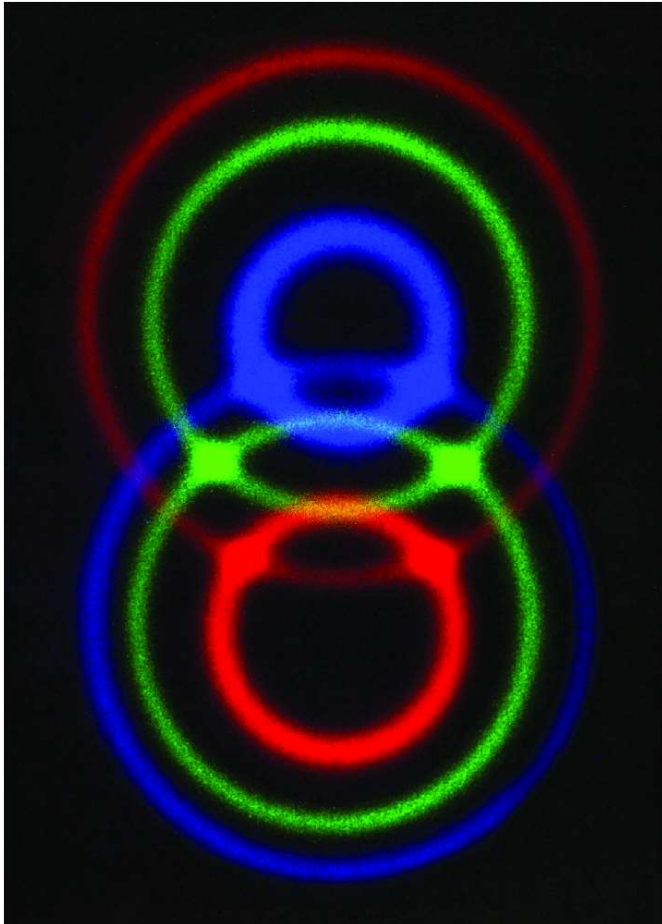
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RESULTS (II):

Twin beam – like correlations

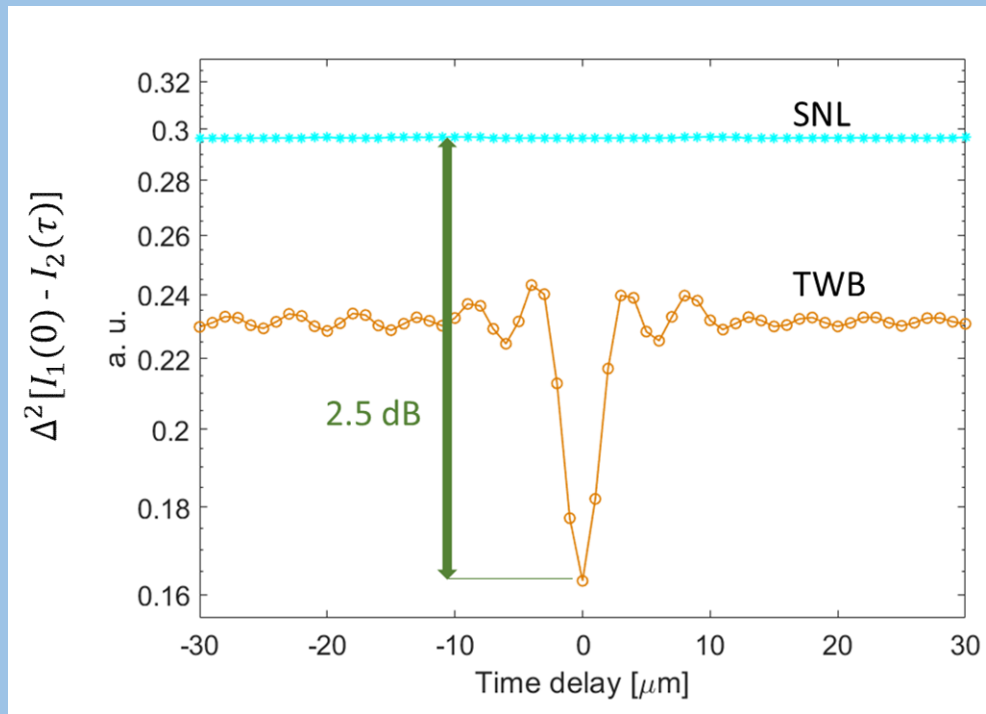
TWIN-BEAM-LIKE STATE



- Quadrature correlation leads to noise reduction in the difference of the photon currents ($\text{NRF} < 1$)

$$\Delta^2[I_1 - I_2] < \text{SNL} = \langle I_1 + I_2 \rangle$$

- 2.5 dB of squeezing measured
- Correlated noise is cancelled
- Uncorrelated noise is detectable below the SNL



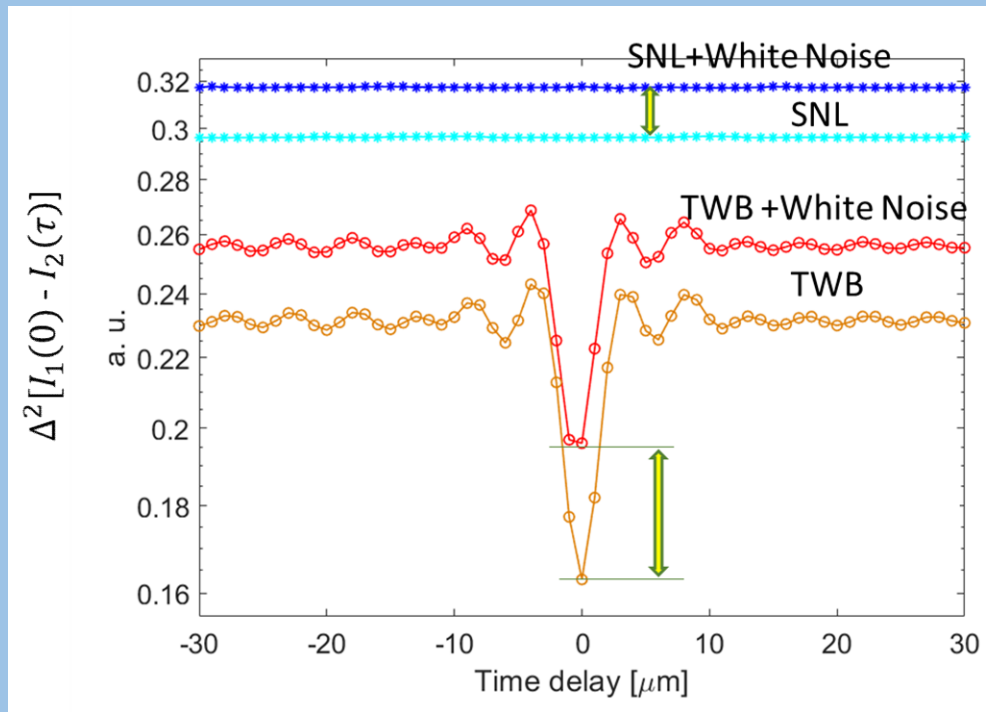
[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph]

Variance of the photocurrent difference in function of the delay time (τ) (TWB)

- Quadrature correlation leads to noise reduction in the difference of the photon currents ($NRF < 1$)

$$\Delta^2[I_1 - I_2] < SNL = \langle I_1 + I_2 \rangle$$

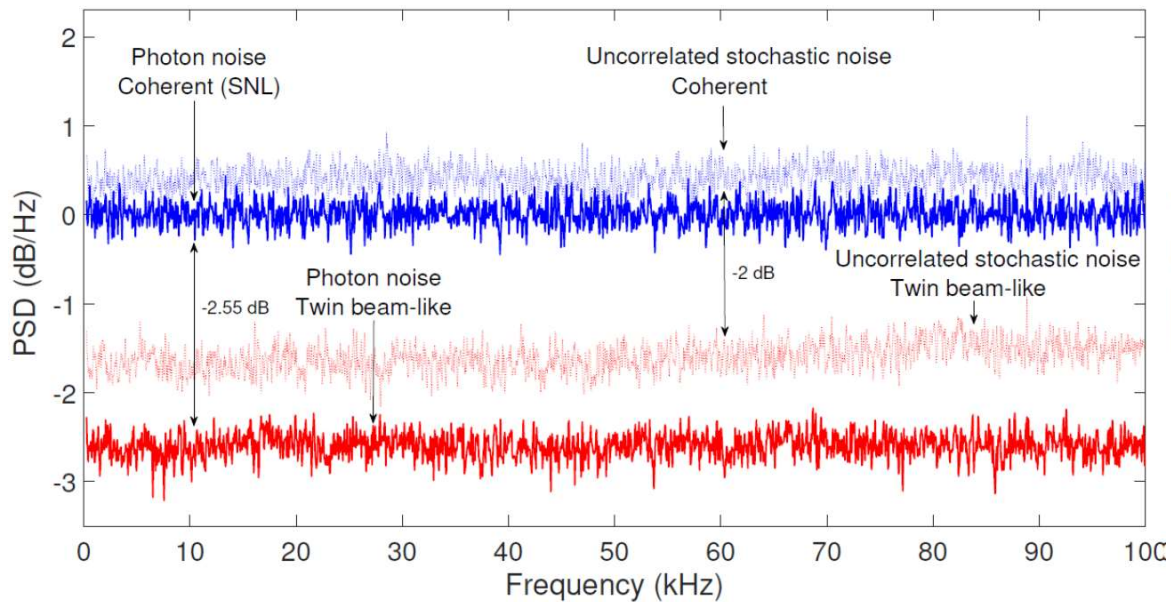
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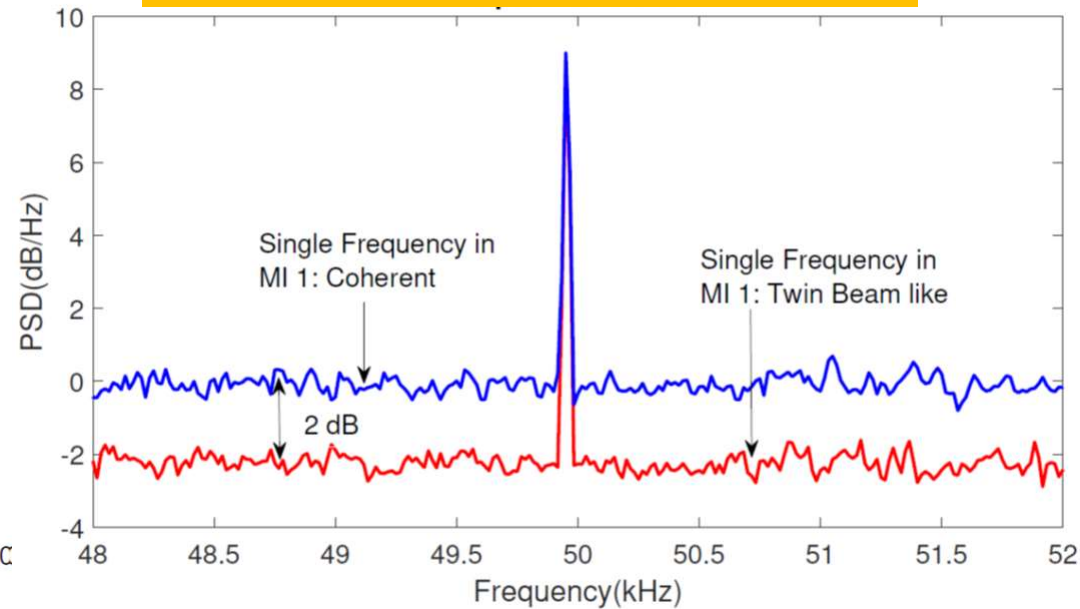
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...same information in the spectral domain

Uncorrelated White noise injected in both interf.s



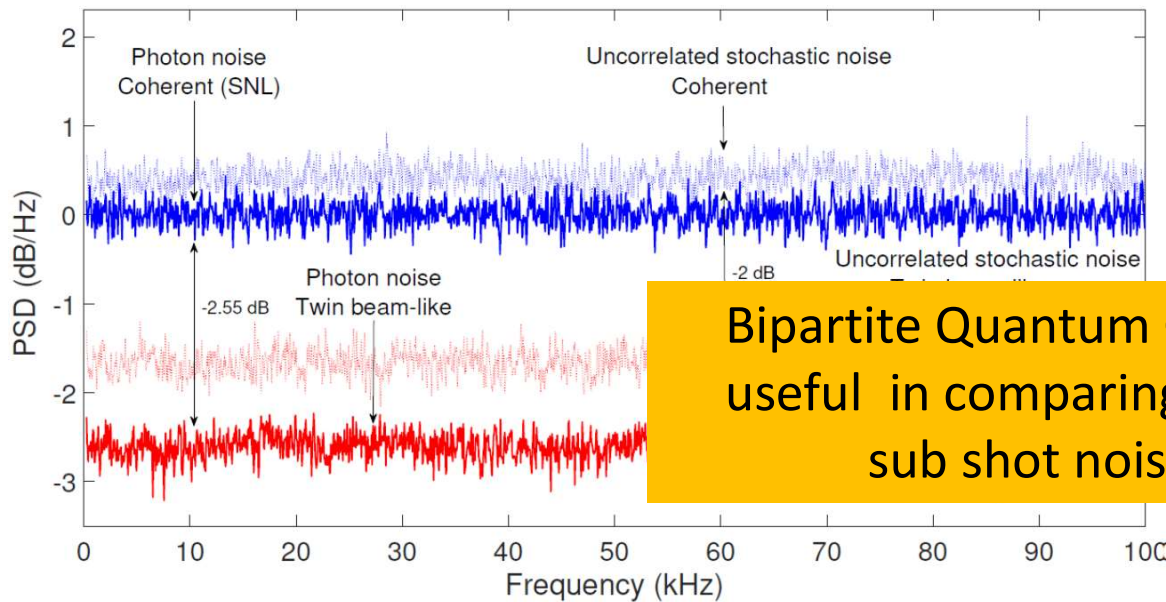
Single frequency noise injected in a one interf.



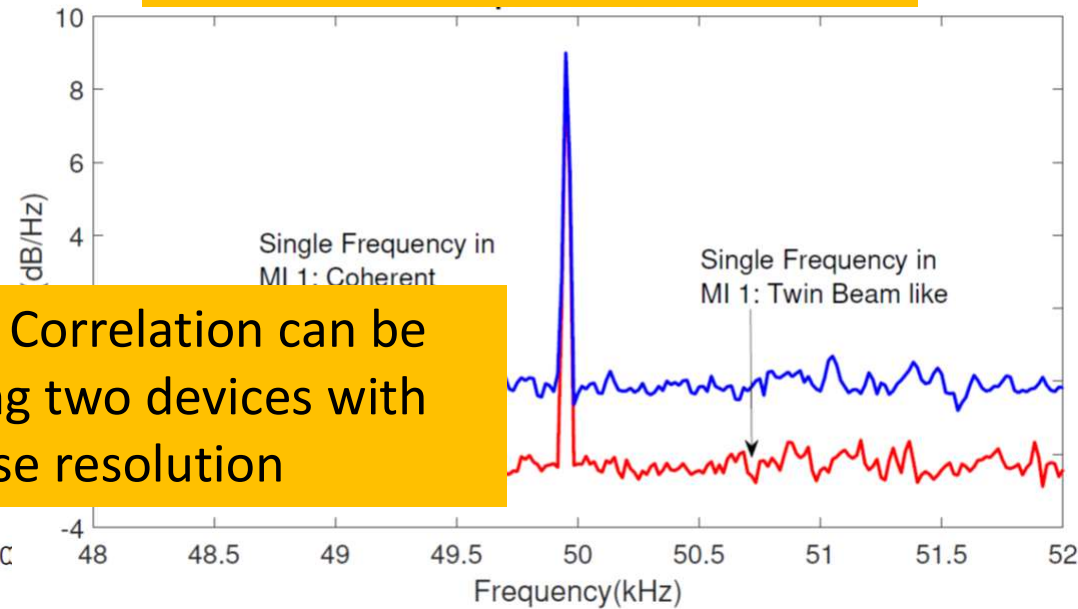
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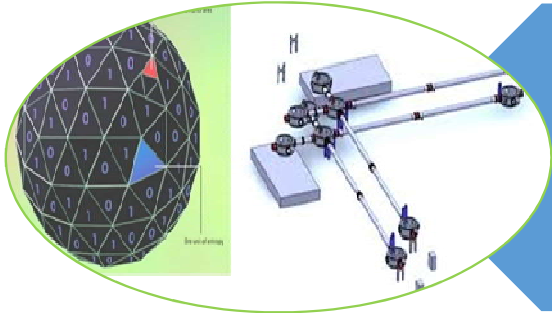


Bipartite Quantum Correlation can be useful in comparing two devices with sub shot noise resolution

[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph]

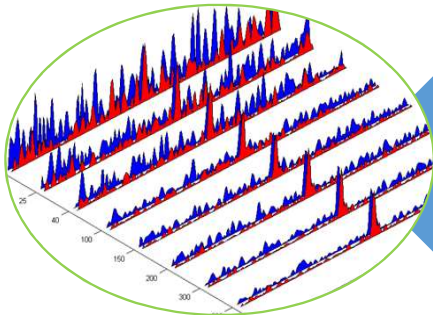
Conclusions & Outlook

Conclusion



Detecting **faint stochastic noises** is important in fundamental physics quests
(*gravitational wave background, Planck scale effects..*)

Correlation techniques boost the sensitivity of the single device of orders of magnitude

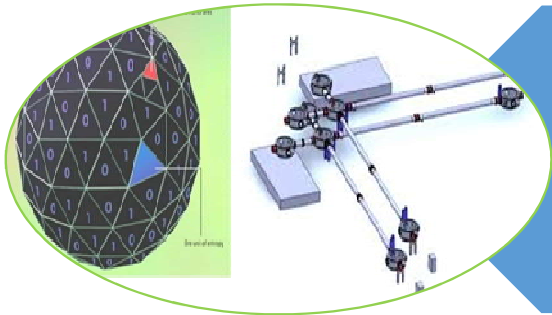


Squeezed light and TWB provide enhancement in comparing signals in two interferometers (TWB could reach in principle disruptive advantage, but challenging in practice)

We have reported a table top experiment mimicking the design of large scale devices demonstrating significant quantum advantage

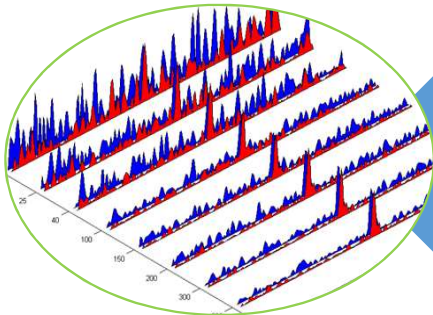
S. T. Pradyumna et al,
*Quantum enhanced correlated interferometry
for fundamental physics tests*
[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph] (2018)

Conclusion



Detecting **faint stochastic noises** is important in fundamental physics quests
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Correlation techniques boost the sensitivity of the single device of orders of magnitude



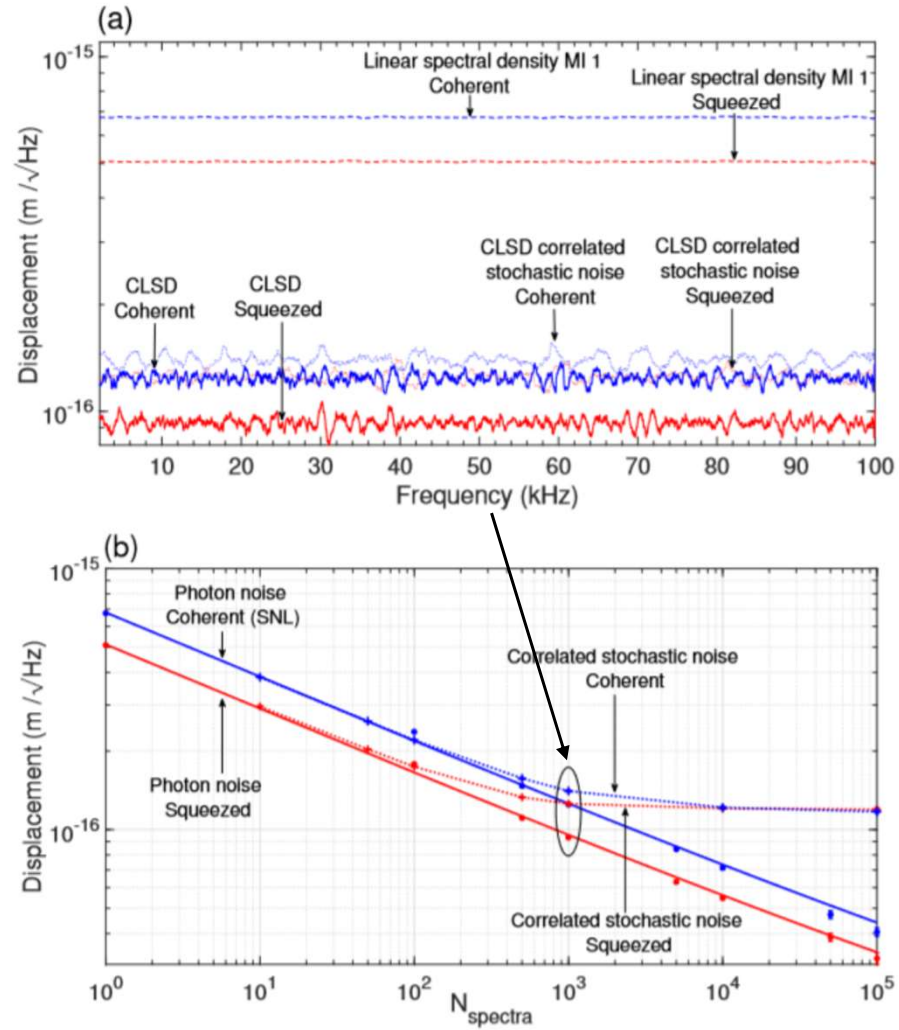
Squeezed light and TWB provide enhancement in comparing signals in two interferometers (TWB could reach in principle disruptive advantage, but challenging in practice)

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Thanks for your attention!

S. T. Pradyumna et al,
*Quantum enhanced correlated interferometry
for fundamental physics tests*
[arXiv:1810.13386v2](https://arxiv.org/abs/1810.13386v2) [quant-ph] (2018)

BACK-UP SLIDES



← Cross Linear Spectral Density

The same stochastic signal was injected in both interferometers, with an amplitude well below the sensitivity of the single MI (approximately 1/5 of the SNL)

Sampling rate: 500 ksample/s
Acquisition time 20 s

While almost a factor of 5.6 of improvement in sensitivity is gained by the cross-spectra statistical averaging, an additional factor of 1.35 is obtained from the injection of squeezed states.

Maximum achieved sensitivity: **3×10^{-17} m/√Hz** (1/20 of SNL)

- Coherent case
- Squeezing injection

$$\delta x(\text{m}/\sqrt{\text{Hz}}) = \frac{\lambda}{2} \frac{V_{\text{rms}}}{V_{\pi}} \frac{1}{\sqrt{BW}}$$



Several QG theories (string theories, holographic theory, heuristic arguments from black holes,...) predict non-commutativity of position variables at Planck scale

$$[\hat{x}_i, \hat{x}_j] = \hat{x}_k \epsilon_{ijk} i c t_P / \sqrt{4\pi}$$

G. Hogan, Arxiv: 1204.5948

G. Hogan, Phys. Rev. D 85, 064007 (2012)

Sort of space-time uncertainty principle (L = radial separation)

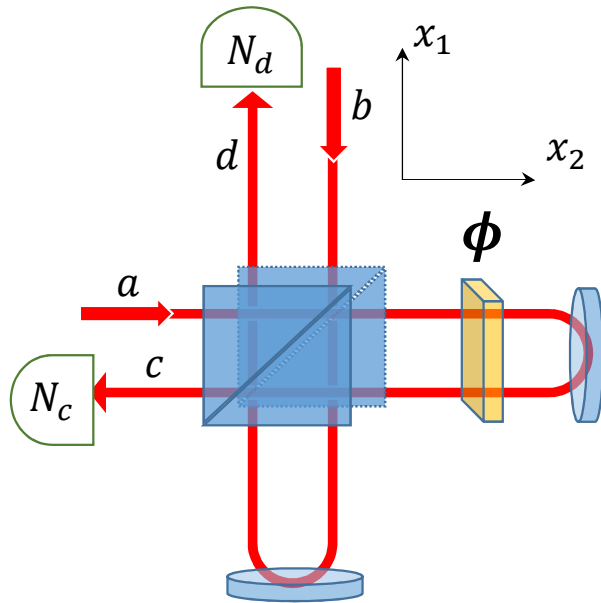
$$\langle \hat{x}_\perp^2 \rangle = L c t_P / \sqrt{4\pi} = (2.135 \times 10^{-18} \text{m})^2 (L/1\text{m})$$

This new **quantum** uncertainty of **space-time** induces a slight random wandering of transverse position (called “**holographic noise**”)

Holometer (**Holo**graphic Interfero**meter**) @ **Fermilab**:
two coupled ultra-sensitive Michelson
interferometers (40 m arms)



<http://holometer.fnal.gov/>



In Michelson interferometer the *phase shift* (ϕ) can be seen as a *simultaneous* measurement of the position of the beam splitter ($x_1 - x_2$).

Holographic noise accumulates as a *random walk* becoming detectable

$$\langle [X(t) - X(t + \tau)]^2 \rangle = c^2 t_P \tau (2/\pi) \quad \tau \ll 2L/c$$

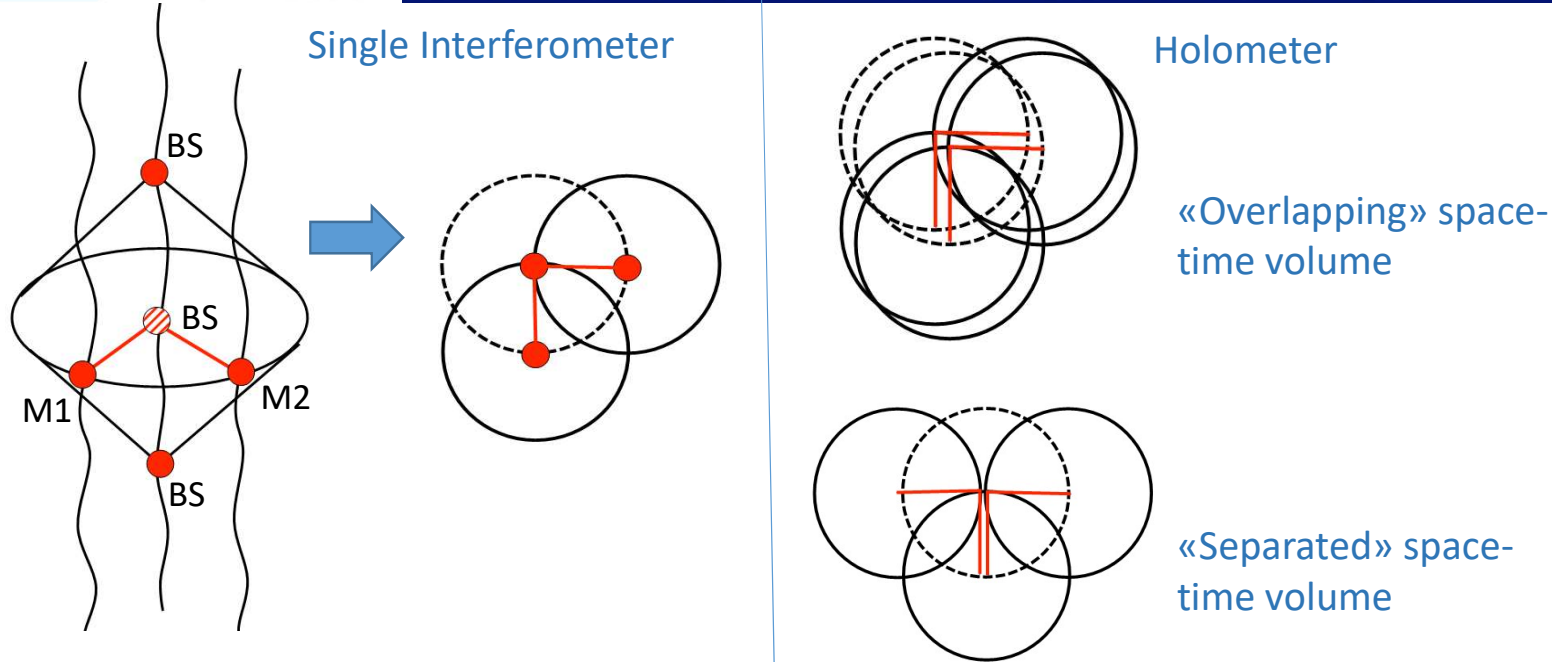
The random walk is bounded (an interferometer measures HN within the *causal boundaries* defined by a *single* light round trip)

($\tau = 2L/c$ the longest time over which differential random walk affects the measured phase)

G. Hogan, Arxiv: 1204.5948

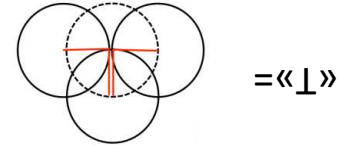
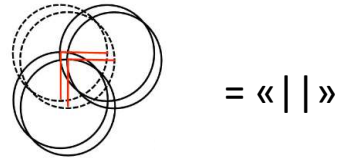
G. Hogan, Phys. Rev. D 85, 064007

(2012)



HOLOMETER: *principles of operation*

- Evaluation of the cross-correlation between two equal Michelson interferometers occupying the same space-time volume
- Reference measurement: HN correlation «turned off» by separating the space-time volumes of the two interferometers



AIM: HN detected by measuring the phase covariance $\mathcal{E}_{\parallel} [\delta\phi_1 \delta\phi_2]$ between the two interferometers of the holometer

$$\delta\phi_k = \phi_k - \phi_{k,0}$$

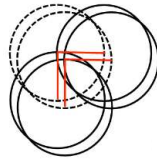
$\hat{C}(\phi_1, \phi_2)$: quantum observable measured at the output of the holometer

$$\mathcal{E}_{\parallel} [\delta\phi_1 \delta\phi_2] \approx \frac{\mathcal{E}_{\parallel} [\hat{C}(\phi_1, \phi_2)] - \mathcal{E}_{\perp} [\hat{C}(\phi_1, \phi_2)]}{\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle}$$

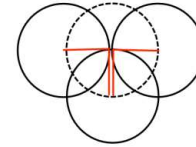
linearization
($\delta\phi_1, \delta\phi_2 \ll 1$)

The uncertainty should be reduced as much as possible

$$U(\delta\phi_1 \delta\phi_2) \approx \sqrt{\frac{\text{Var}_{\parallel} [\hat{C}(\phi_1, \phi_2)] + \text{Var}_{\perp} [\hat{C}(\phi_1, \phi_2)]}{[\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle]^2}}$$



= « || »



= « ⊥ »

Phases covariance uncertainty

$$\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\text{Var}_{\parallel} [\hat{C}(\phi_1, \phi_2)] + \text{Var}_{\perp} [\hat{C}(\phi_1, \phi_2)]}{[\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle]^2}}$$

$$\text{Var}_x [\hat{C}(\phi_1, \phi_2)] \equiv \mathcal{E}_x [\hat{C}^2(\phi_1, \phi_2)] - \mathcal{E}_x [\hat{C}(\phi_1, \phi_2)]^2$$

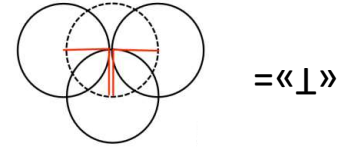
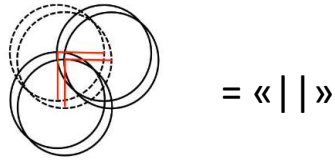
$$\mathcal{E}_x [\hat{O}(\phi_1, \phi_2)] \equiv \int \langle \hat{O}(\phi_1, \phi_2) \rangle f_x(\phi_1, \phi_2) d\phi_1 d\phi_2$$

$f_x(\phi_1, \phi_2)$ pdf of phase fluctuations due to HN
 $x = \parallel, \perp$

Quantum EV
 $\text{Tr}[\rho_{12} \hat{C}(\phi_1, \phi_2)]$

- $f_{\perp}(\phi_1, \phi_2) = \mathcal{F}_{\perp}^{(1)}(\phi_1) \mathcal{F}_{\perp}^{(2)}(\phi_2)$
- $\mathcal{F}_{\parallel}^{(k)}(\phi_k) = \mathcal{F}_{\perp}^{(k)}(\phi_k)$





Phases covariance uncertainty $\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\text{Var}_{\parallel} [\hat{C}(\phi_1, \phi_2)] + \text{Var}_{\perp} [\hat{C}(\phi_1, \phi_2)]}{\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle}^2}$

linearization ($\delta\phi_1, \delta\phi_2 \ll 1$)

$\text{Var}_x [\hat{C}(\phi_1, \phi_2)] = \text{Var} [\hat{C}(\phi_{1,0}, \phi_{2,0})] + \sum_k A_{kk} \mathcal{E}_x [\delta\phi_k^2] + A_{12} \mathcal{E}_x [\delta\phi_1\delta\phi_2] + \mathcal{O}(\delta\phi^3)$

- 0-th order independent from PSs fluctuations (i.e. HN)
- 0-th order quantum light noise (shot-noise in the actual Holometer)

0-th order contribution to PSs covariance unc.: $\mathcal{U}^{(0)} = \frac{\sqrt{2 \text{Var} [\hat{C}(\phi_{1,0}, \phi_{2,0})]}}{|\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle|}$

$\text{Var} [\hat{C}(\phi_{1,0}, \phi_{2,0})] = \langle \hat{C}(\phi_{1,0}, \phi_{2,0})^2 \rangle - \langle \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle^2$

Exploiting quantum light to beat the “shot-noise” level!

PRL 110, 213601 (2013)



Phys. Rev. Lett. 117, 111102 (2016)

Search for Space-Time Correlations from the Planck Scale with the Fermilab Holometer

Aaron S. Chou,^a Richard Gustafson^b, Craig Hogan^{a,c}, Brittany Kamai^{c,g}, Ohkyung Kwon^{c,e}, Robert Lanza^{c,d}, Lee McCuller^{c,d}, Stephan S. Meyer^c, Jonathan Richardson^c, Chris Stoughton^e, Raymond Tomlin^e, Samuel Waldman^f, Rainer Weiss^d

^a *Fermi National Accelerator Laboratory;*

^b *University of Michigan;*

^c *University of Chicago;*

^d *Massachusetts Institute of Technology;*

^e *Korea Advanced Institute of Science and Technology (KAIST);*

^f *SpaceX;*

^g *Vanderbilt University*

Measurements are reported of high frequency cross-spectra of signals from the Fermilab Holometer, a pair of co-located 39 m, high power Michelson interferometers. The instrument obtains differential position sensitivity to cross-correlated signals far exceeding any previous measurement in a broad frequency band extending to the 3.8 MHz inverse light crossing time of the apparatus. A model of universal exotic spatial shear correlations that matches the Planck scale holographic information bound of space-time position states is excluded to 4.6σ significance.

Squeezed light in gravitational wave detectors!!

A sub-shot-noise PS measurement in a **single** interferometer (e.g. gravitational wave detector) was suggested exploiting squeezed light

Caves, PRD 23, 1693 (1981)

Kimble et al., PRD 65, 022002 (2001)

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

PRL 110, 213601 (2013)

PHYSICAL REVIEW LETTERS

week ending
24 MAY 2013

Quantum Light in Coupled Interferometers for Quantum Gravity Tests

I. Ruo Berchera,¹ I. P. Degiovanni,¹ S. Olivares,² and M. Genovese¹

¹INRIM, Strada delle Cacce 91, I-10135 Torino, Italy

²Dipartimento di Fisica, Università degli Studi di Milano, and CNISM UdR Milano Statale, Via Celoria 16, I-20133 Milano, Italy
(Received 22 January 2013; published 21 May 2013)

PHYSICAL REVIEW A 92, 053821 (2015)

One- and two-mode squeezed light in correlated interferometry

I. Ruo-Berchera,¹ I. P. Degiovanni,¹ S. Olivares,^{2,3} N. Samantaray,^{1,4} P. Traina,¹ and M. Genovese^{1,5}



Does squeezed light help also in the case of the Holometer?

$\hat{C}(\phi_1, \phi_2)$ is the covariance of photon # differences

$$\hat{C}(\phi_1, \phi_2) = \Delta \hat{N}_{1-}(\phi_k) \Delta \hat{N}_{2-}(\phi_k)$$

$$\Delta \hat{N}_{k-}(\phi_k) = \hat{N}_{k-}(\phi_k) - \mathcal{E}[\hat{N}_{k-}(\phi_k)]$$

$$\hat{N}_{-}(\phi) = \hat{N}_c(\phi) - \hat{N}_d(\phi)$$



0-th order contribution to PSs covariance unc.:

$$\mathcal{U}^{(0)} = \frac{\sqrt{2 \text{Var}[\hat{C}(\phi_{1,0}, \phi_{2,0})]}}{|\langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle|} = \sqrt{2} \frac{\lambda + \mu (1 + 2\lambda - 2\sqrt{\lambda + \lambda^2})}{(\lambda - \mu)^2} \quad (\phi_0 = \frac{\pi}{2})$$

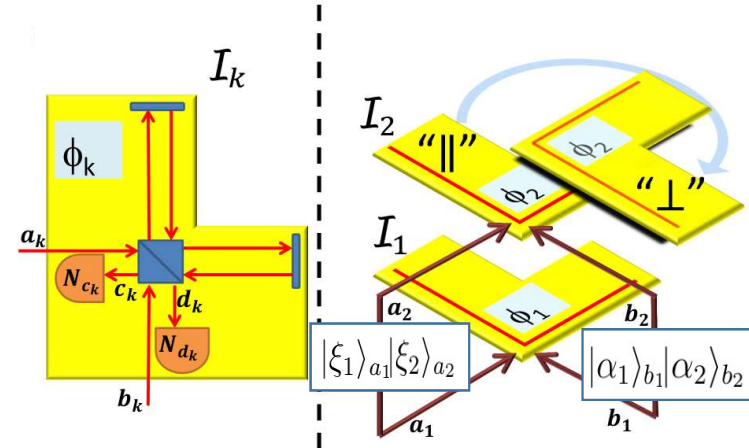


$$\mathcal{U}_{\text{SQ}}^{(0)} \approx (2\sqrt{2}\lambda\mu)^{-1}$$

$\mu \gg \lambda \gg 1$

μ : mean # photons coherent light
 λ : mean # photons squeezed light

i.e. $(4\lambda)^{-1}$ better than the CL case $\mathcal{U}_{\text{CL}}^{(0)} \approx \sqrt{2}/\mu$



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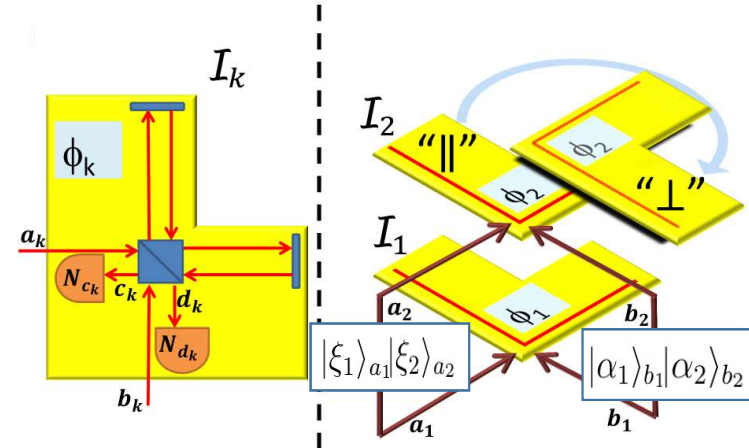
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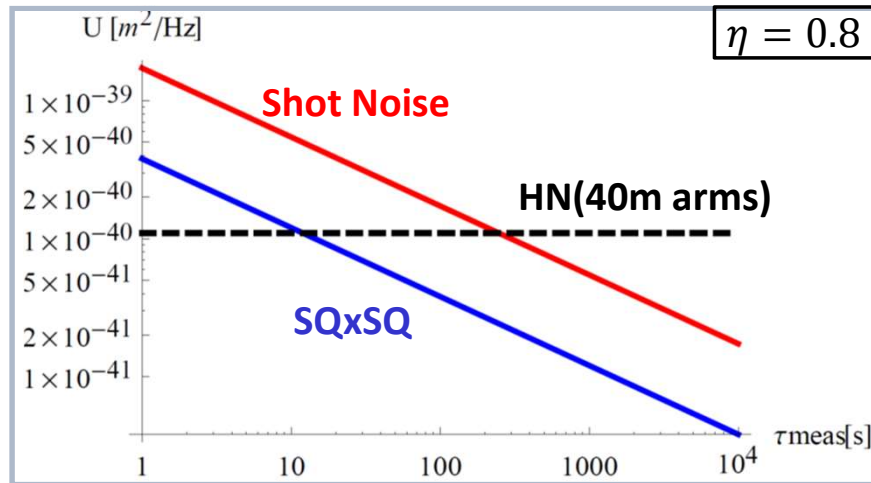
In the presence of losses η :

$$u_{\text{SQ}}^{(0)} / u_{\text{CL}}^{(0)} \approx (1 - \eta) + \eta / (4\lambda)^1$$

$$\mu \gg \lambda \gg 1$$

$$u_{\text{SQ}}^{(0)} / u_{\text{CL}}^{(0)} \approx 1 - 2\eta\sqrt{\lambda}$$

$$\lambda \ll 1 \text{ and } \mu \gg 1$$



For SQXSQ, we expect a reduction of the measurement time of more than one order of magnitude

- $\lambda_{opt} = 1064$ nm.

- $\mu = 10, \eta = 0.8$.

- $P_{opt} = 2000$ W

$\tau_{sample} = 2L/c,$

Does quantum correlated light help in coupled interferometers?

Twin-Beam light in the a 's ports: $|\text{TWB}\rangle\rangle_{a_1, a_2} = S_{12}(\zeta)|0\rangle_{a_1, a_2}$

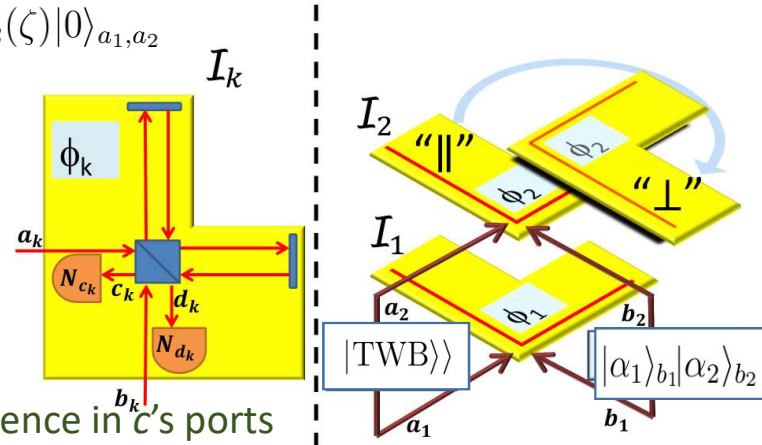
$$S_{12}(\zeta) = \exp(\zeta a_1^\dagger a_2^\dagger - \zeta^* a_1 a_2)$$

Coherent light in the b 's ports: $|\alpha_k\rangle_{b_k} = D_{b_k}(\alpha_k)|0\rangle_{b_k}$

$$D_{b_k}(\alpha_k) = \exp(\alpha_k b_k^\dagger - \alpha_k^* b_k)$$

$\hat{C}(\phi_1, \phi_2)$ is the fluctuations of the photon # difference in c 's ports

$$\hat{C}(\phi_1, \phi_2) = \Delta^2 [\hat{N}_{c_1} - \hat{N}_{c_2}]$$



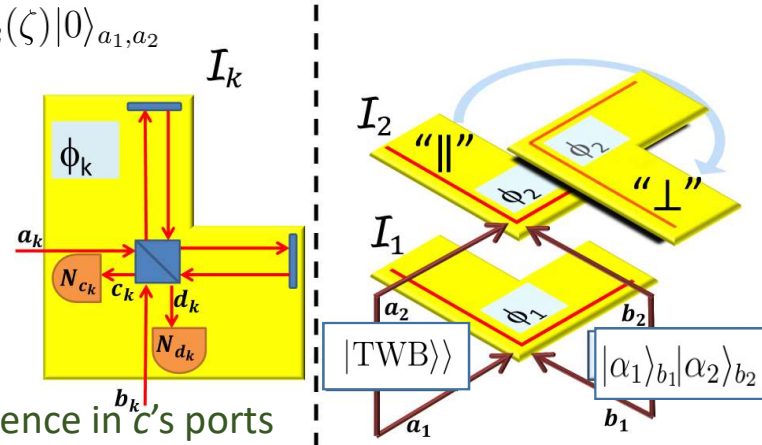
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$$\phi_{k,0} = 0 \implies \mathcal{U}_{\text{TWB}}^{(0)} = 0$$

$$\langle\langle \text{TWB} | (\hat{N}_{a_1} - \hat{N}_{a_2})^M | \text{TWB} \rangle\rangle = 0, \forall M > 0$$

$$\implies \text{Var} \left\{ \Delta^2 [\hat{N}_{c_1}(0) - \hat{N}_{c_2}(0)] \right\} = 0$$

$$\left| \langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle \right| = -\frac{1}{2} \sqrt{\lambda(1+\lambda)} \mu \cos[2(\theta_\zeta - \theta_\alpha)]$$



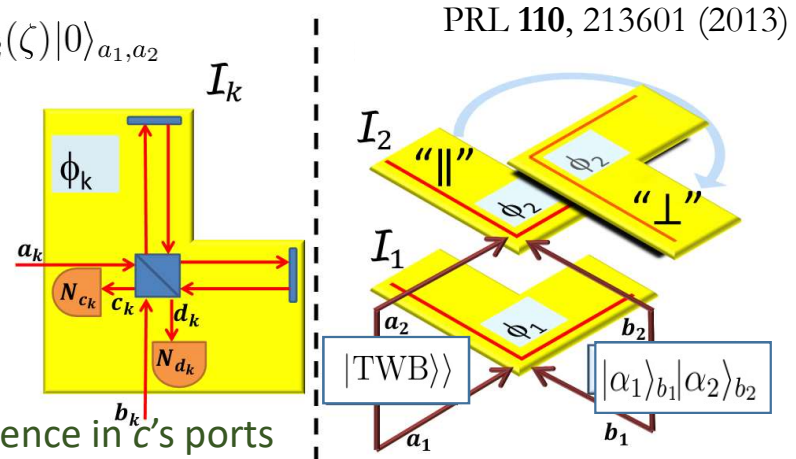
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$\phi_{k,0} = 0 \rightarrow \mathcal{U}_{\text{TWB}}^{(0)} = 0$

$\rightarrow \text{Var} \left\{ \Delta^2 [\hat{N}_{c_1}(0) - \hat{N}_{c_2}(0)] \right\} = 0$
 $\left| \langle \partial_{\phi_1, \phi_2}^2 \hat{C}(\phi_{1,0}, \phi_{2,0}) \rangle \right| = -\frac{1}{2} \sqrt{\lambda(1+\lambda)} \mu \cos[2(\theta_\zeta - \theta_\alpha)]$

In the presence of losses η :

$\lambda \ll 1$ and $\mu \gg 1$

$$\mathcal{U}_{\text{TWB}}^{(0)} / \mathcal{U}_{\text{CL}}^{(0)} \approx \sqrt{2(1-\eta)/\eta}$$

(Squeezed light)

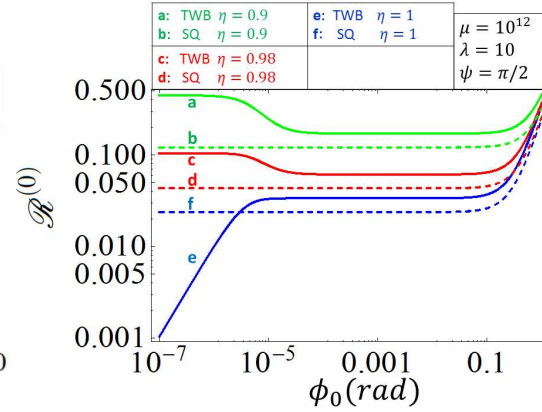
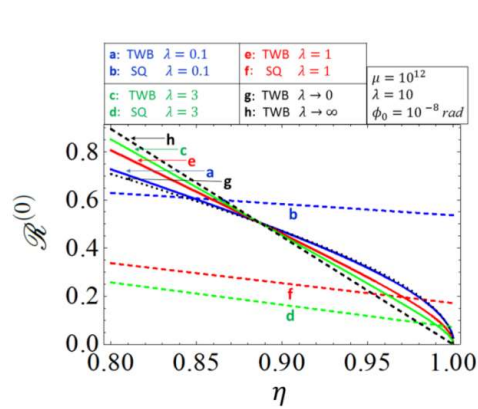
$$\mathcal{U}_{\text{SQ}}^{(0)} / \mathcal{U}_{\text{CL}}^{(0)} \approx 1 - 2\eta\sqrt{\lambda}$$

$\mu \gg \lambda \gg 1$

$$\mathcal{U}_{\text{TWB}}^{(0)} / \mathcal{U}_{\text{CL}}^{(0)} \approx 2\sqrt{5}(1-\eta)$$

(Squeezed light)

$$\mathcal{U}_{\text{SQ}}^{(0)} / \mathcal{U}_{\text{CL}}^{(0)} \approx (1-\eta) + \eta/(4\lambda)$$

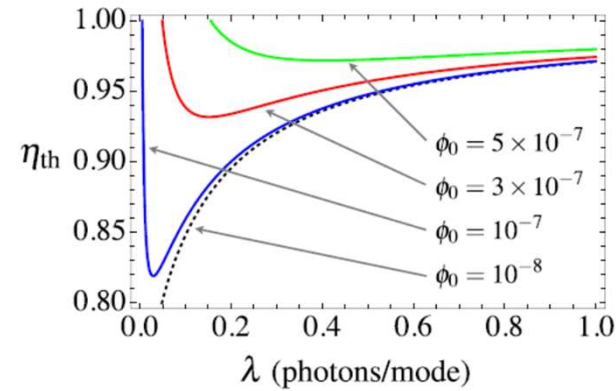


Quantum
Enhancement
Uncertainty

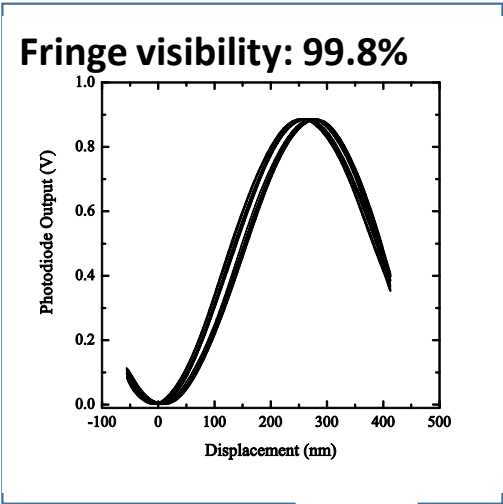
$$\approx \sqrt{2(1 - \eta)/\eta}$$



$$\approx 0 \quad (\eta \approx 1)$$

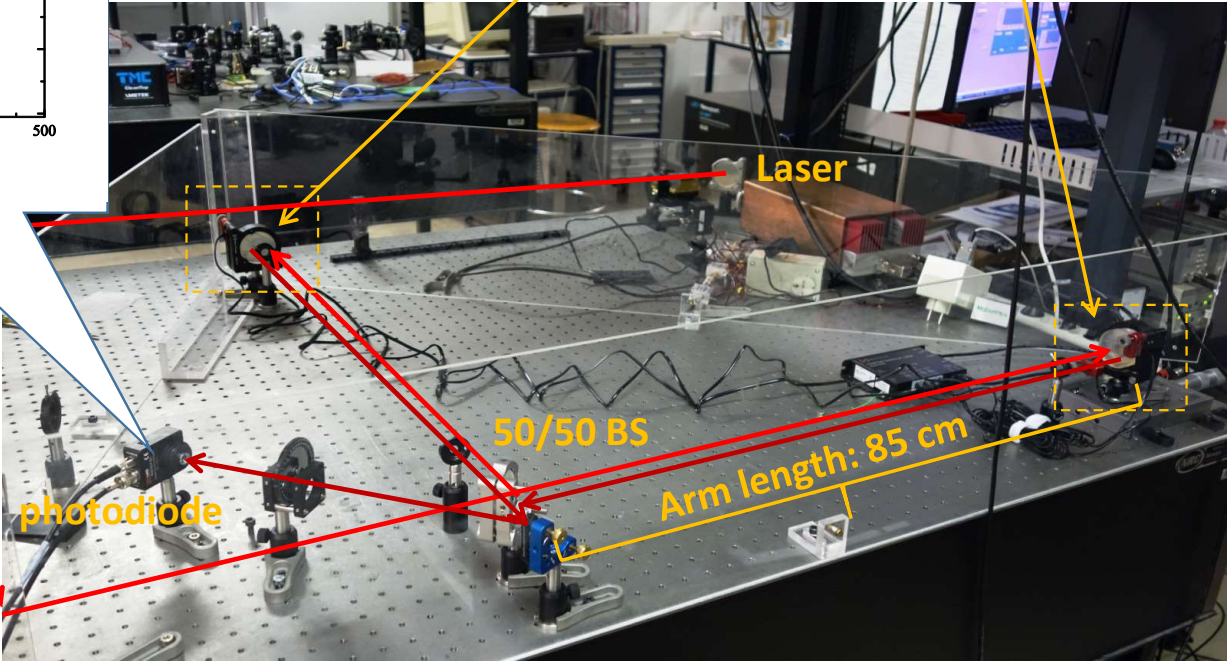
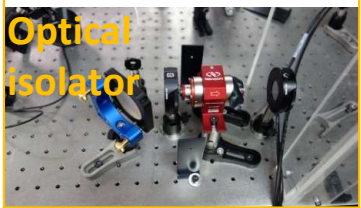
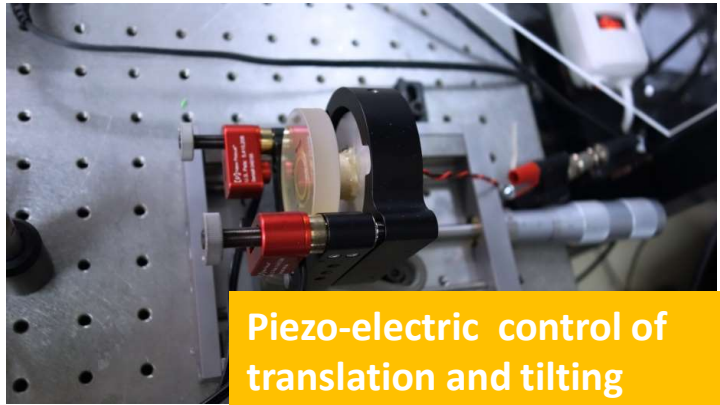


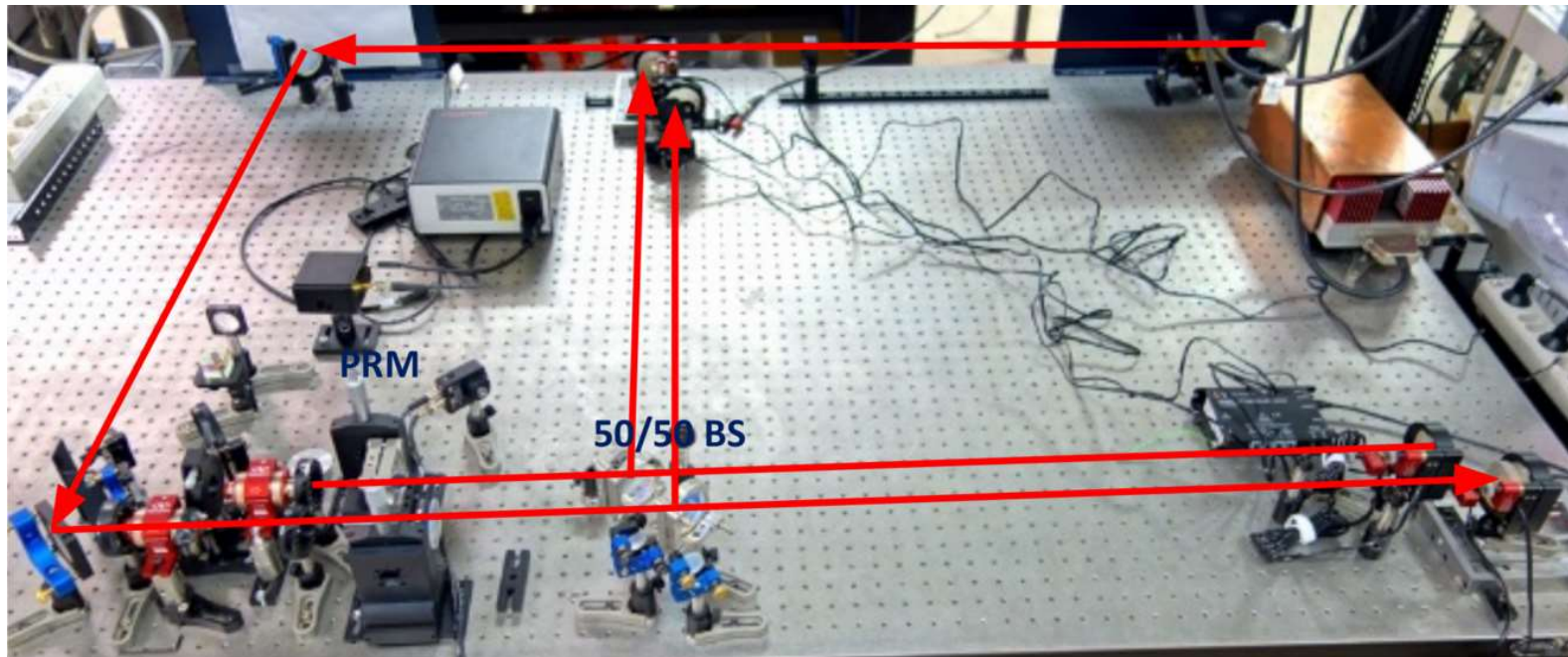
Michelson Interferometer : Preliminary Results



High-Reflectance end-mirrors

$R > 99.8\%$



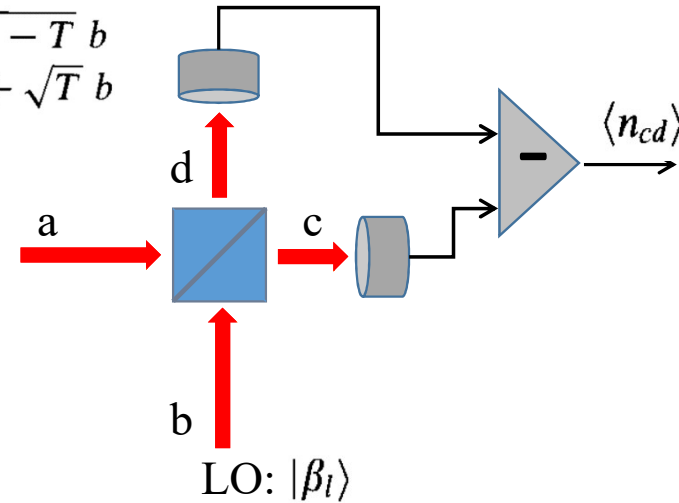


How to measure Quadratures

BS transformation:

$$c = \sqrt{T} a + i\sqrt{1-T} b$$

$$d = i\sqrt{1-T} a + \sqrt{T} b$$



50:50 BS:

$$n_{cd} = c^\dagger c - d^\dagger d = -i(a^\dagger b - b^\dagger a)$$



$$\langle n_{cd} \rangle = -2|\beta_l| \langle X(\phi_l + \pi/2) \rangle$$

$$(\Delta n_{cd})^2 = 4|\beta_l|^2 [\Delta X(\phi_l + \pi/2)]^2$$

$$X(\phi) \equiv X_\phi = \frac{1}{2}(ae^{-i\phi} + a^\dagger e^{i\phi})$$



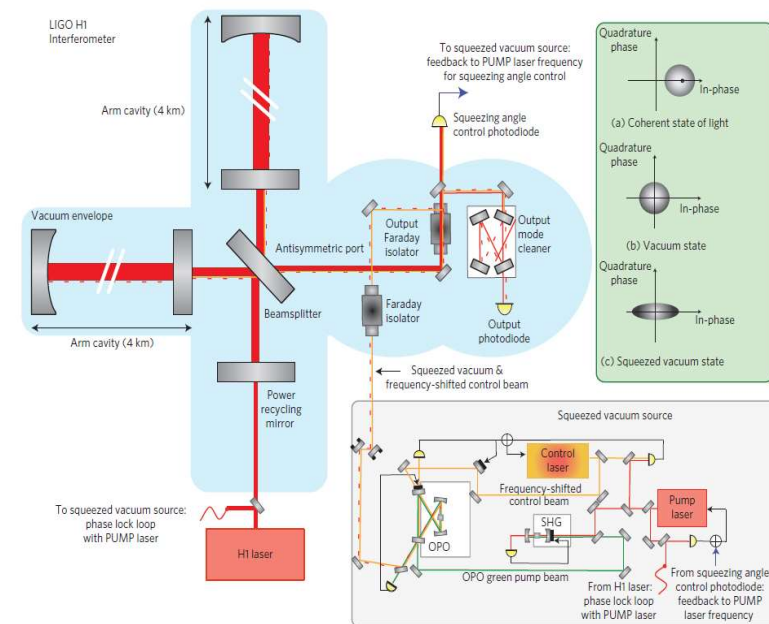
Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

$$S_{12}(\zeta)|0\rangle$$

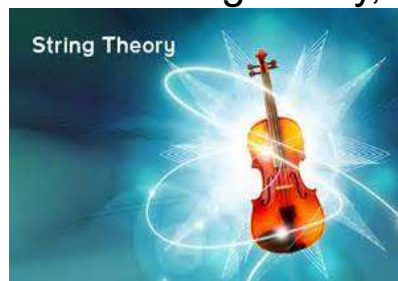
$$S_{12}(\zeta) = \exp(\zeta a_1^\dagger a_2^\dagger - \zeta^* a_1 a_2)$$

2.15 ± 0.05 dB improvement



An experimental quantum gravity?

- ❑ The dream of building a theory unifying general relativity and quantum mechanics, the so called quantum gravity has been a key element in theoretical physics research for the last 60 years.
- ❑ A HUGE theoretical work: string theory, loop gravity,



- ❑ However, for many years no testable prediction emerged from these studies. In the last few years this common wisdom was challenged: a first series of testable proposals concerned photons propagating on cosmological distances [AmelinoCamelia et al.], with the problem of extracting QG effects from a limited (uncontrollable) observational sample affected by various propagation effects.



- The TWB state is expressed as:

$$\rho_{TWB}[\varphi] = \sum_{n,m} (1-t^2)t^{n+m} \text{Exp}[i\varphi(n-m)] |n\rangle|n\rangle\langle m|\langle m| \quad t = \tanh(\zeta)$$

- We generate a mixed state ρ_{mix} that preserves the photon number correlation, simulating a Gaussian dephasing

$$\rho_{mix} = \int d\varphi \rho_{TWB} \frac{e^{-\frac{\varphi^2}{2\delta^2}}}{\sqrt{2\pi\delta^2}}$$

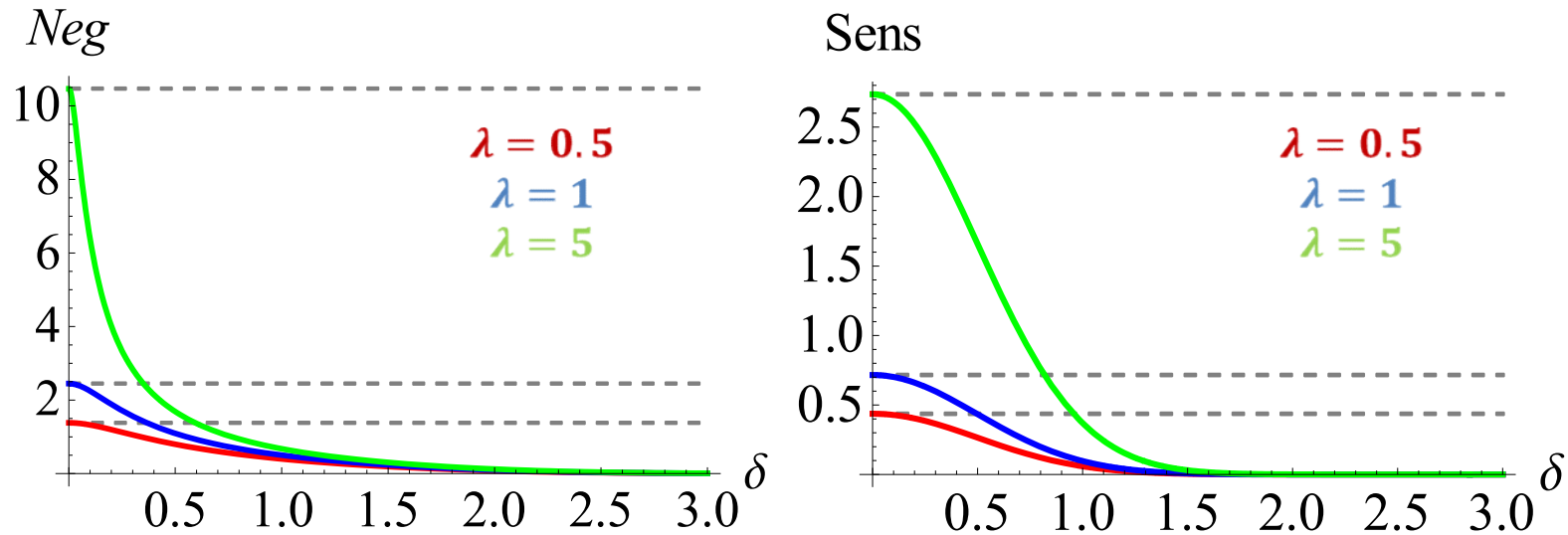
For $\delta=0$ we have $\rho_{mix} = \rho_{TWB}$,

for $\delta=2\pi$ we have $\rho_{mix} = \sum_n (1-t^2)t^{2n} |n\rangle|n\rangle\langle n|\langle n|$

- We studied the negativity and the sensitivity coefficient in function of the amplitude of the dephasing



Is Entanglement related to the TWB quantum enhancement?



Indeed a clear role of entanglement, measured by negativity [see M.Roncaglia,A.Montorsi, M.G. Phys. Rev A 90, 062303 (2014)], is demonstrated. This is due to the fact that the scheme requires not only perfect photon number correlation, but also a defined phase of the TWB for a coherent interference with the classical coherent field at the Beam Splitter.

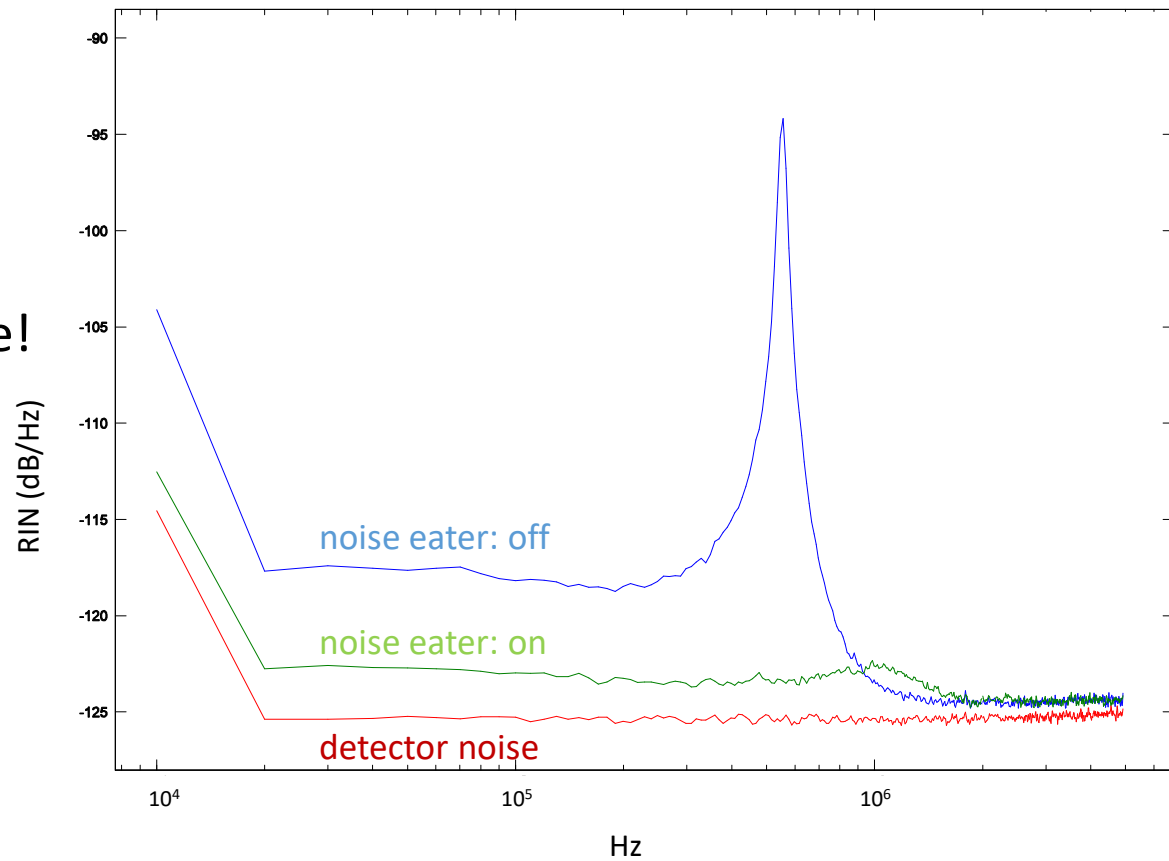


The Laser

COHERENT MEPHISTO (cw)
Nd:YAG @ 1064 nm
Output power up to 2 W



Extremely low noise!



Does Q-correlated (Entangled) light help in coupled interferometers?

Twin-Beam state (or Two-mode squeezed vacuum)

Hamiltonian of a non-degenerate parametric process: $H \propto a^\dagger b^\dagger + h.c.$

(Unitary) Two-mode “Squeeze” Operator : $S_2(\xi) = \exp \{ \xi a^\dagger b^\dagger - \xi^* ab \}$ $\xi = r e^{i\psi}$

$$S_2^\dagger(\xi) \begin{pmatrix} a \\ b^\dagger \end{pmatrix} S_2(\xi) = \mathbf{S}_{2\xi} \begin{pmatrix} a \\ b^\dagger \end{pmatrix}$$

$$\mathbf{S}_{2\xi} = \begin{pmatrix} \mu & \nu \\ \nu^* & \mu \end{pmatrix}$$

$$\mu = \cosh r$$

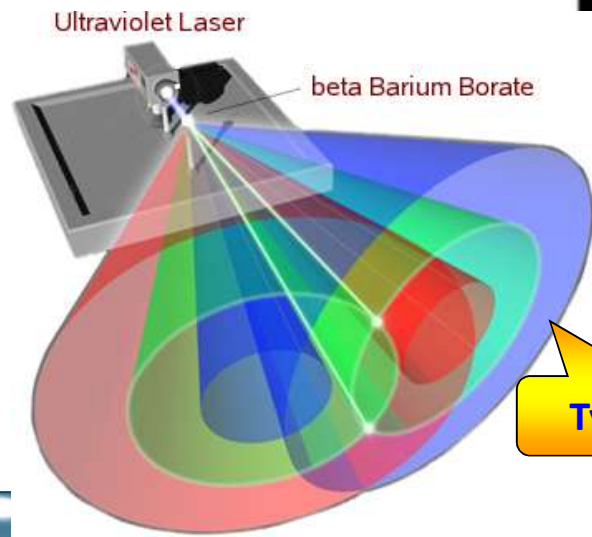
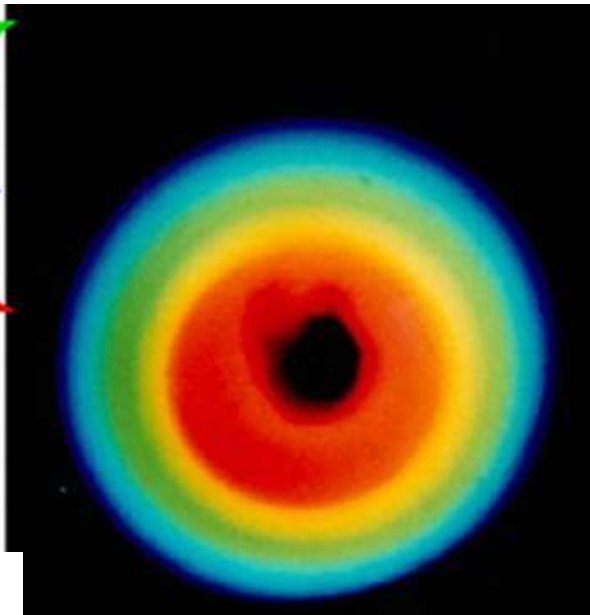
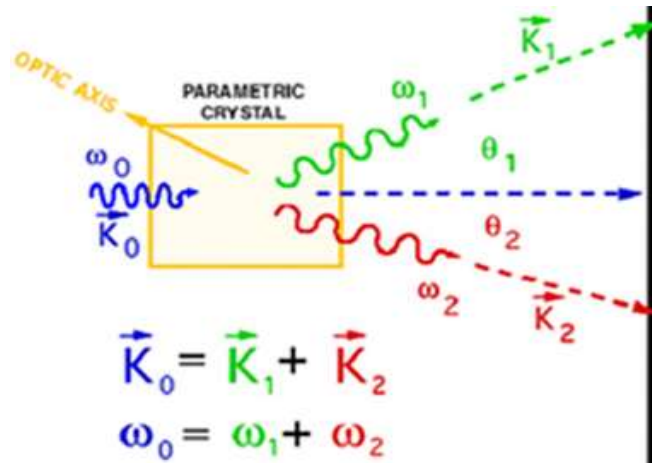
$$\nu = e^{i\psi} \sinh r$$

Twin Beam state: $|\text{TWB}\rangle\rangle = S_2(\xi)|\mathbf{0}\rangle = \frac{1}{\sqrt{\mu}} \sum_{k=0}^{\infty} \left(\frac{\nu}{\mu} \right)^k |k\rangle \otimes |k\rangle$

TWB shows **perfect correlation** in the **photon number**, i.e TWB is an eigenstate of the photon number difference



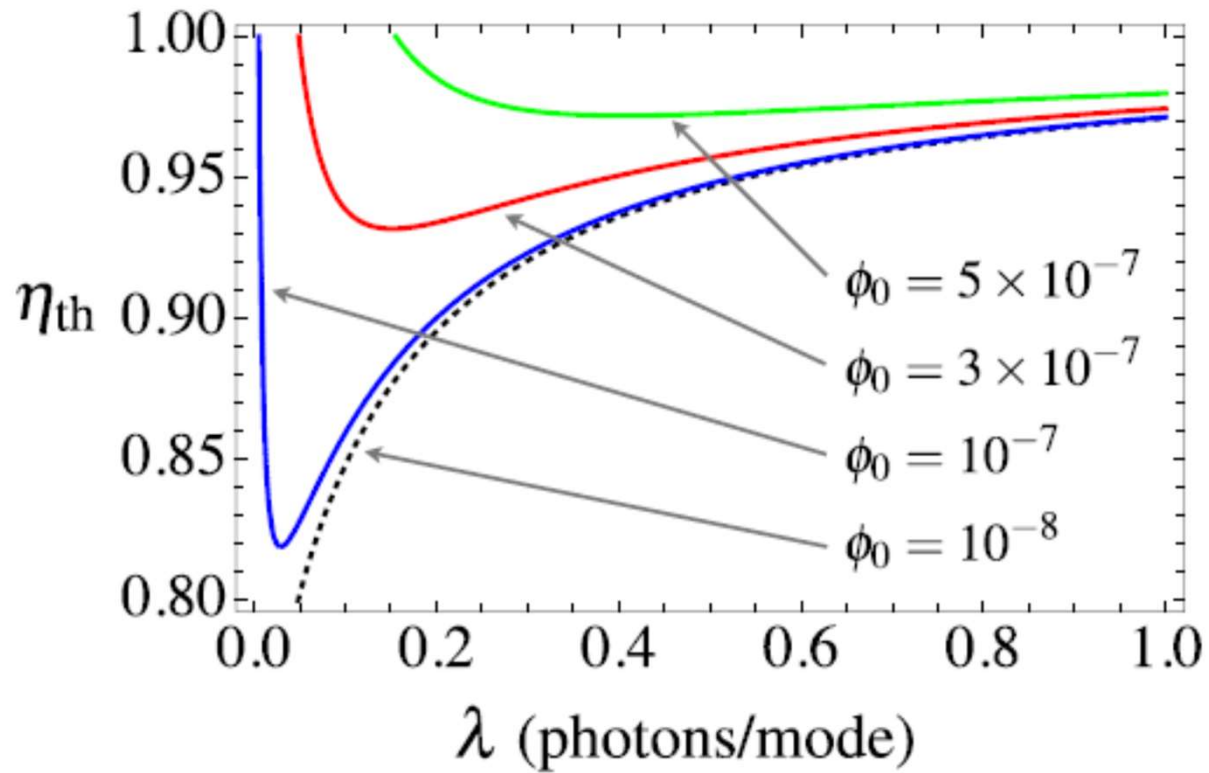
PDC: a brief summary



Type-II PDC

Type-I PDC





Squeezed light in gravitational wave detectors!!

A sub-shot-noise PS measurement in a **single** interferometer (e.g. gravitational wave detector) was suggested exploiting squeezed light

Caves, PRD 23, 1693 (1981)

Kimble et al., PRD 65, 022002 (2001)

...

and recently realized at Ligo 600

R. Schnabel et al., Nature Commun. 1, 121 (2010)

Ligo, Nature Phys. 7, 962 (2011)

Does squeezed light help also in the case of the Holometer?

Squeezed light in the a 's ports: $|\xi_k\rangle_{a_k} = S_{a_k}(\xi_k)|0\rangle_{a_k}$

$$S_{a_k}(\xi_k) = \exp[\xi_k (a_k^\dagger)^2 - \xi_k^* (a_k)^2]$$

Coherent light in the b 's ports: $|\alpha_k\rangle_{b_k} = D_{b_k}(\alpha_k)|0\rangle_{b_k}$

$$D_{b_k}(\alpha_k) = \exp(\alpha_k b_k^\dagger - \alpha_k^* b_k)$$

