Cosmology from group field theory

Edward Wilson-Ewing

University of New Brunswick

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Motivation

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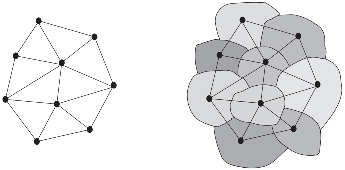
In loop quantum cosmology (LQC), the symmetries of the space-time of interest (e.g., FLRW) are imposed classically, then a loop-like quantization is performed on the remaining degrees of freedom (e.g., a, π_a, ϕ, π_ϕ). The result is that the big-bang singularity is replaced by a non-singular bounce.

Despite this success, the exact relation between loop quantum cosmology and full loop quantum gravity remains unclear. It is important to go beyond LQC, using any hints LQC may offer.

Loop Quantum Gravity: Basics

Loop quantum gravity is a background independent approach to quantum gravity based on connection and triad variables. [Ashtekar; Immirzi; Barbero]

A convenient basis for states in the canonical framework are spin-networks: graphs coloured by spins on the edges and intertwiners on the nodes. [Penrose]



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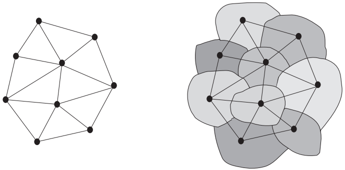

[Rovelli]

An important result is that geometrical observables like the volume and the area have a discrete spectrum. Furthermore, each node can be thought of as a polyhedron with some volume and surface areas transverse to the links: for example, a four-valent node gives a tetrahedron. In this sense, the spin-network is composed of 'atoms of geometry'. [Rovelli, Smolin; Ashtekar, Lewandowski; Freidel, Speziale; Bianchi, Dona, Speziale; Haggard; . . .]

Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of Planck-scale quanta of geometry, it is reasonable to assume that in cosmological space-times:

- there are many quanta of geometry,
- one quantum contributes a small fraction of the spatial volume,
- cosmological expansion is primarily due to quanta being added.

Cosmological Expansion

Cosmological expansion could occur through two different processes (or a combination of them):

1. increasing the number of quanta of geometry, keeping the volume of each individual quantum constant,
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The universe has expanded by at least a factor of $\sim 10^{28}$; if the original discreteness scale was $\ell_{\rm Pl}$ then, assuming possibility 2, the discreteness scale today should be at least $10^{28}\ell_{\rm Pl} \sim 10^{-7}$ m.

Since there is no observational evidence of discreteness today, this suggests that the first possibility is the dominant process.

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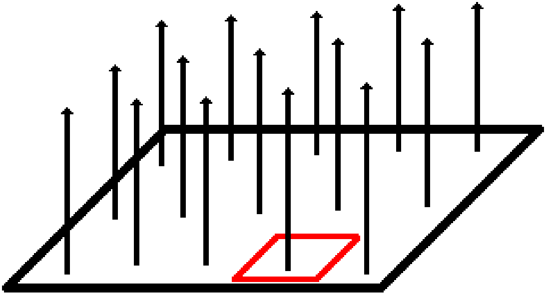
Furthermore, LQC suggests that all N quanta of geometry are in fact in the same state corresponding to the minimal excitation of geometry possible, and thus the total spatial volume at some time is

$$V_{tot} = N V_{min}.$$

Insight from Loop Quantum Cosmology

A key step in LQC is constructing the field strength operator from the holonomy of the Ashtekar-Barbero connection A_a^i around a minimal loop [Ashtekar, Pawłowski, Singh; Ashtekar, WE; ...].



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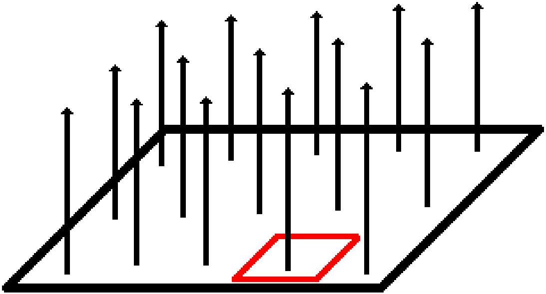

The area of this loop is taken to be the minimal area in LQG. Heuristically, this assumes that the dominant contribution to the area of a surface comes from these minimal area excitations.

And these minimal area excitations are all in the same state: this suggests **a condensate of geometry** [Gielen, Oriti, Sindoni, WE].

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$$V_{tot} = NV_{min}.$$

It is easiest to construct condensate states starting from a field theory with creation and annihilation operators; this leads to **group field theory**, a field theory for the quanta of geometry of LQG.

Cosmology: Hydrodynamics of the Condensate

The key idea here is that the continuous cosmological space-time emerges from the coarse-graining/hydrodynamics of the group field theory (GFT) condensate state.

The microscopic dynamics of the GFT condensate state imply some effective coarse-grained Friedmann equations:

> Find the relevant collective cosmological observables (e.g., total spatial volume) and calculate their dynamics for this condensate state, as determined by the microscopic GFT model (with respect to some relational time).

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> Find the relevant collective cosmological observables (e.g., total spatial volume) and calculate their dynamics for this condensate state, as determined by the microscopic GFT model (with respect to some relational time).

Note that we will make assumptions on the type of LQG/GFT state that is relevant for cosmology, but we do not impose any symmetries upon the underlying GFT theory.

Outline

1. Group Field Theory
2. Condensate States
3. Cosmological Dynamics

Covariant Loop Quantum Gravity

The covariant approach to loop quantum gravity (spin foams) gives a prescription to calculate transition amplitudes between an 'initial' and a 'final' quantum state using a sum-over-histories approach.

A possible basis for the initial and final quantum states is given by spin-networks. It is possible to interpret a spin-network as a many-particle state, with each spin-network node and the 'half-links' emanating from it corresponding to one (group-valued) 'particle' or 'quantum of geometry'.

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Non-trivial dynamics occur when spin-network nodes meet and interact. Using this langauge, a spin foam model provides the Feynman rules for the allowed interactions between spin-network nodes in the theory.

Group Field Theory

It is possible to construct a field theory with an action $S[\varphi]$ such that the perturbative expansion of the partition function [De Pietri, Freidel, Krasnov, Rovelli; Reisenberger, Rovelli]

$$Z = \int \mathcal{D}\varphi \, e^{-S[\varphi]}$$

gives the Feynman rules for any spin foam model, with φ being the field whose excitations are spin-network nodes; as a result φ lives on a group manifold and such a theory is called a group field theory (GFT).

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Here I will take φ defined on $SU(2)^{\times 4} \times \mathbb{R}$,

$$\varphi := \varphi^{j_1,j_2,j_3,j_4,\iota}_{m_1,m_2,m_3,m_4}(\chi).$$

The j_i and m_i are the $SU(2)$ representations on the links of the (four-valent) spin network node and the intertwiner ι captures the gauge-invariance at the node. The matter field χ, which here I take to be a massless minimally-coupled scalar field, will act as a relational

Group Field Theory Action

The simplest GFT action for quantum gravity (coupled to a massless scalar field χ) has the form where, schematically, the dominant terms are [Oriti, Sindoni, WE; Li, Oriti, Zhang]

$$S \sim \sum_{j_i, m_i, \iota_i} \int_\chi \left[\varphi \, K_2^{(0)} \varphi + \varphi \, K_2^{(2)} \partial_\chi^2 \varphi \right] + \lambda \sum_{j_i, m_i, \iota_i} \int_\chi \mathcal{V}_5(j_i, m_i, \iota_i) \, \varphi^5 .$$

It turns out that this action is local in χ, but the interaction term is non-local in j_i, m_i, ι_i.

While the GFT action was chosen so it gives a field theory whose Feynman rules are those of a spin foam model, it is possible to use other techniques to study the quantum theory. Here, I will take the Legendre transform of the Lagrangian to construct a relational Hamiltonian, with the scalar field χ playing the role of time.

A Relational Hamiltonian

The Legendre transform gives

$$\mathcal{H}[\varphi] = -\sum_{j,m,\iota} \left[\frac{\pi_{\vec{m}}^{\vec{\jmath},\iota}(\chi)^2}{2\, K^{(2)}_{\vec{\jmath},\vec{m},\iota}} + K^{(0)}_{\vec{\jmath},\vec{m},\iota} \, \frac{\varphi_{\vec{m}}^{\vec{\jmath},\iota}(\chi)^2}{2} \right] + U[\varphi],$$

with $\pi_{\vec{m}}^{\vec{\jmath},\iota}(\chi) = -K^{(2)}_{\vec{\jmath},\vec{m},\iota} \partial_\chi \varphi_{\vec{m}}^{\vec{\jmath},\iota}(\chi)$ the momentum conjugate to φ and $U[\varphi]$ the potential term.

Defining the 'equal relational time' Poisson brackets

$$\{\varphi_{\vec{m}_1}^{\vec{\jmath}_1,\iota_1}(\chi), \pi_{\vec{m}_2}^{\vec{\jmath}_2,\iota_2}(\chi)\} = \delta^{\vec{\jmath}_1,\vec{\jmath}_2} \delta_{\vec{m}_1,\vec{m}_2} \delta^{\iota_1,\iota_2},$$

the (classical) equations of motion for any observable $\mathcal{O}$ in the GFT can be derived from

$$\frac{d\mathcal{O}}{d\chi} = \{\mathcal{O}, \mathcal{H}\}.$$

The Quantum Theory

The quantum theory can be constructed by replacing the equal time Poisson bracket by commutation relations, in the Schrödinger picture

$$\left[\, \hat{\varphi}^{\vec{\jmath}_1,\iota_1}_{\vec{m}_1},\, \hat{\pi}^{\vec{\jmath}_2,\iota_2}_{\vec{m}_2} \,\right] = i\hbar\, \delta^{\vec{\jmath}_1,\vec{\jmath}_2} \delta_{\vec{m}_1,\vec{m}_2} \delta^{\iota_1,\iota_2},$$

and then the relational Schrödinger equation for GFT wave functionals is

$$\hat{\mathcal{H}}\, \Psi = i\hbar\, \frac{d\Psi}{d\chi}.$$

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$$\hat{\mathcal{H}}\,\Psi = i\hbar\,\frac{d\Psi}{d\chi}.$$

Since the Hamiltonian contains terms that are quadratic in the field and momentum operators, it is convenient to introduce creation and annihilation operators, e.g.,

$$\hat{a}^{\dagger}_{\vec{\jmath},\vec{m},\iota} = \frac{1}{\sqrt{2\hbar\,\omega^{\vec{\jmath},\iota}_{\vec{m}}}} \left(\omega^{\vec{\jmath},\iota}_{\vec{m}}\, \hat{\varphi}^{\vec{\jmath},\iota}_{\vec{m}} - i\,\pi^{\vec{\jmath},\iota}_{\vec{m}}\right), \qquad \omega^{\vec{\jmath},\iota}_{\vec{m}} = \sqrt{|K^{(0)}_{\vec{\jmath},\vec{m},\iota}\, K^{(2)}_{\vec{\jmath},\vec{m},\iota}|},$$

with $[\,\hat{a}_{\vec{\jmath}_1,\vec{m}_1,\iota_1},\, \hat{a}^{\dagger}_{\vec{\jmath}_2,\vec{m}_2,\iota_2}\,] = \delta_{\vec{\jmath}_1,\vec{\jmath}_2}\delta_{\vec{m}_1,\vec{m}_2}\delta_{\iota_1,\iota_2}$; GFT states live in a bosonic Fock space.

The Relational Hamiltonian

If $K^{(0)}_{\vec{\jmath},\vec{m},\iota}$ and $K^{(2)}_{\vec{\jmath},\vec{m},\iota}$ have the opposite signs, then

$$\hat{\mathcal{H}} = \frac{\hbar}{2} \sum_{j,m,\iota} M_{\vec{\jmath},\vec{m},\iota} \left((\hat{a}^{\dagger}_{\vec{\jmath},\vec{m},\iota})^2 + \hat{a}^2_{\vec{\jmath},\vec{m},\iota} \right) + U[\hat{\varphi}],$$

with

$$M_{\vec{\jmath},\vec{m},\iota} = \sqrt{\left| \frac{K^{(0)}_{\vec{\jmath},\vec{m},\iota}}{K^{(2)}_{\vec{\jmath},\vec{m},\iota}} \right|}.$$

Note that we expect $K^{(0)}_{\vec{\jmath},\vec{m},\iota}$ and $K^{(2)}_{\vec{\jmath},\vec{m},\iota}$ to have opposite signs in order for there to be an instability in the theory so that the universe can expand indefinitely.

Condensate States

Recall that we are interested in condensate states for cosmology. The simplest family of condensate states are the Gross-Pitaevskii condensate states: coherent states of the GFT field operator [Gielen, Oriti, Sindoni]. In the Heisenberg picture,

$$|\sigma\rangle \sim \exp\left(\sum_{j_i,m_i,\iota} \sigma^{j_i,\iota}_{m_i}\, \hat{a}^{\dagger}{}^{j_i,\iota}_{m_i}(\chi_o)\right)|\mathbf{0}\rangle,$$

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Appropriate approximations will simplify the equations to be solved. In other contexts, the Gross-Pitaevskii condensate approximation is typically most trustworthy when interactions are small. Thus, we will focus on the regime where the interaction term is negligible.

The Small Interactions Regime

Considering small interactions appears reasonable for the spatially flat FLRW space-time. The main observable is the total volume where connectivity is unimportant, and the space-time is spatially flat so we do not need to worry about encoding the spatial curvature in the connectivity of the graph [Gielen, Oriti, Sindoni].

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If the interaction term $U[\varphi]$ becomes important, then it will likely be necessary to go beyond the Gross-Pitaevskii approximation and change the form of the GFT state in order to include the connectivity information between GFT excitations. A quick calculation indicates that this is expected to happen at large volumes (depending on the coupling constants in the theory).

Observables

The key observable of interest in cosmology is the spatial volume, which in GFT is given by

$$\hat{V} = \sum_{j,m,\iota} V_{\vec{\jmath},\iota} a^{\dagger}_{\vec{\jmath},\vec{m},\iota} a_{\vec{\jmath},\vec{m},\iota},$$

where $V_{\vec{\jmath},\iota}$ denotes the eigenvalue of the LQG volume operator acting on a spin-network node with the quantum numbers $\vec{\jmath}, \iota$.

For $|\sigma\rangle$,

$$\langle\sigma|a_{\vec{\jmath},\vec{m},\iota}(\chi_o)|\sigma\rangle = \sigma^{\vec{\jmath},\iota}_{\vec{m}},$$

so $\langle\hat{V}(\chi_o)\rangle_\sigma = \sum_{j,m,\iota} V_{\vec{\jmath},\iota}|\sigma^{\vec{\jmath},\iota}_{\vec{m}}|^2$.

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The momentum π_χ of the scalar field χ, related to the energy density by $\varepsilon = \pi_\chi^2/2V^2$, is given by $\hat{\pi}_\chi = i\hbar\partial_\chi$ which corresponds to the GFT relational energy.

Dynamics

At the initial relational time χ_o,

$$\langle \frac{\mathrm{d}\hat{a}_{\vec{\jmath},\vec{m},\iota}}{\mathrm{d}\chi} \rangle_\sigma = \frac{1}{i\hbar} \langle [a_{\vec{\jmath},\vec{m},\iota}, \hat{\mathcal{H}}] \rangle_\sigma = -iM_{\vec{\jmath},\vec{m},\iota} \langle a^\dagger_{\vec{\jmath},\vec{m},\iota} \rangle_\sigma = -iM_{\vec{\jmath},\vec{m},\iota} \, \bar{\sigma}^{\vec{\jmath},\iota}_{\vec{m}}.$$

Assuming that this relation continues to hold, which is expected if interactions are negligible and the Gross-Pitaevskii form of the condensate state is preserved, then

$$\langle \hat{V}(\chi) \rangle_\sigma = \sum_{j,m,\iota} V_{\vec{\jmath},\iota} |\langle a_{\vec{\jmath},\vec{m},\iota}(\chi) \rangle_\sigma|^2,$$

and the dynamics of the state are given by

$$\frac{\mathrm{d}\langle a_{\vec{\jmath},\vec{m},\iota}(\chi) \rangle_\sigma}{\mathrm{d}\chi} = -iM_{\vec{\jmath},\vec{m},\iota} \, \langle a_{\vec{\jmath},\vec{m},\iota}(\chi) \rangle^*_\sigma,$$

with * indicating complex conjugation. These dynamics, derived in the limit that the interaction term is negligible, can be solved exactly.

Cosmological Observables

The solution gives

$$\langle \hat{V} \rangle_\sigma = \sum_{j,m,\iota} V_{\vec{\jmath},\iota} A_{\vec{\jmath},\vec{m},\iota} \cosh\Big(2 M_{\vec{\jmath},\vec{m},\iota} (\chi - \tilde{\chi}^o_{\vec{\jmath},\vec{m},\iota})\Big),$$

where $A_{\vec{\jmath},\vec{m},\iota} > 0$ and $\tilde{\chi}^o_{\vec{\jmath},\vec{m},\iota}$ are constants of integration. Clearly, the spatial volume never vanishes and the big-bang singularity is replaced by a non-singular bounce.

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This can be compared with the solution to the Friedmann equation, expressed using χ as a relational clock, $V = A \exp[\sqrt{12\pi G}(\chi - \chi_o)]$, to see that the correct classical limit is obtained for $M^2_{\vec{\jmath},\vec{m},\iota} = 3\pi G$.

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Also, for the matter sector, a short calculation gives

$$\langle \pi_\chi \rangle = \hbar \sum_{j,m,\iota} M_{\vec{\jmath},\vec{m},\iota} A_{\vec{\jmath},\vec{m},\iota} = \mathrm{constant}.$$

This matches the classical result $\dot{\pi}_\chi = 0$.

Relation to LQC

LQC, in its construction, suggests that the appropriate condensate state is one where all the quanta are equilateral spin-network nodes with $j = 1/2$. Motivated by this observation, let's consider the case where $\sigma_j(\chi)$ only has support on one excitation $\hat{a}_o^\dagger$.

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Then, the sums trivialize and (setting $M_o^2 = 3\pi G$) we find $\langle \pi_\chi \rangle_\sigma = \sqrt{3\pi G}\, \hbar A_o$ and

$$\langle \hat{V} \rangle_\sigma = \frac{V_o \langle \pi_\chi \rangle_\sigma}{\sqrt{3\pi G}\, \hbar} \cosh\Big(\sqrt{12\pi G} (\chi - \tilde{\chi}_o) \Big),$$

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For sharply-peaked states in LQC,

$$\langle V \rangle_{LQC} = \frac{\pi_\chi}{\sqrt{2\varepsilon_c}} \cosh\left(\sqrt{12\pi G}(\chi - \chi_o)\right), \qquad \varepsilon_c \sim \frac{1}{G^2 \hbar}.$$

Conclusions

- Motivated by simple arguments combined with insights from LQC, we made a specific ansatz on the type of state in (the GFT reformulation of) LQG that corresponds to cosmological space-times: GFT condensate states.
- Using a relational formulation of GFT, Gross-Pitaevskii states are found to have simple dynamics in the limit that interactions are negligible. In particular, for these states the dynamics for the spatial volume show that the big-bang singularity is replaced by a non-singular bounce, and the correct classical limit is recovered for some choices of parameters in the GFT action.
- The dynamics of sharply-peaked states in LQC are exactly recovered for condensates where one type of quantum excitation of geometry dominates the condensate, with the identification $\varepsilon_c = 3\pi G\hbar^2/2V_o^2$.

Outlook

There are many open questions, including:

- Study other condensate wave functions and GFT actions [Gielen; Pithis, Sakellariadou; Adjei, Gielen, Wieland],
- Details of the semi-classical regime at late times [Gielen],
- Allow for scalar fields with non-vanishing potentials $V(\chi)$ and other types of matter content [Li, Oriti, Zhang],
- Include spatial curvature and anisotropies [de Cesare, Oriti, Pithis, Sakellariadou],
- Develop perturbation theory and determine its relation to cosmological perturbation theory [Gielen, Oriti; Gerhardt, Oriti, WE],
- Study the regime of large interactions [de Cesare, Pithis, Sakellariadou, Tomov],
- Include connectivity information in the analysis,
- And more. . .

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Thank you for your attention!